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FLEXIBILITY OF PRE-SERVICES TEACHERS IN PROBLEM POSING

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The present research examines mathematics pre-service teachers' flexibility in problem posing in four environments: with technology and with 'What if not' strategy, with technology but without 'What if not' strategy, without technology but with 'What if not' strategy, and without technology and without 'What if not'. Seventy-nine pre-service teachers participated in the research. The research findings show that the participants used three posing approaches that included six strategies and posed ten problem types. The group, who used technology and 'What if not' strategy, used all the six strategies and raised all types of problems. It can be concluded that the combination between technology and 'What if not' strategy affected positively the participants' flexibility in problem posing.

INTRODUCTION

Recently problem posing is attracting the attention of mathematics educators because of its potentialities to affect positively the problem solving capacities of students, as well as other aspects of students' learning; for example their critical and creative thinking. Moreover, creative thinking is one aspect of mathematics learning that mathematics education researchers have been paying attention to as enriching the learning process (e.g., Silver, 1997). Though defining creativity is not a simple task, researchers describe it as the ability to think conceptually. One of the definitions of creativity includes describing its three components: fluency, flexibility and originality (Guilford, 1975; Leikin, Koichu, & Berman, 2009; Torrance, 1974). This research is interested in flexibility.

Studies on mathematics creativity report its positive influence on students' learning, for example their appreciation of the beauty of mathematics, as well as developing their talents (Mann, 2006). Two studies that investigated students' problem posing in the frame of the Paper Pool activity, used also in the present research, are those of Silver, Mamona–Downs, Leung, and Kenny (1996) and of Kontorovich, Koichu, Leikin, and Berman (2011). The first study results showed that the participating pre-service and in-service teachers posed problems in two manners: keeping problem givens fixed and varying the givens. The second research results related to flexibility showed that the participating high achieving secondary school students, working in groups, used three problem posing strategies: (1) Accepting the givens; (2) varying the givens; and (3) introducing new types of givens. At the same time, the participants raised three types of problems: (1) Analyzing the ball path; (2) finding the ball path (s) under given constraints; and (3) "static" geometrical problems. We intend to study pre-service teachers' flexibility when posing problems in the Paper Pool activity. What distinguishes the present research is the four environments in which the participants worked: with technology and with 'What if not' strategy, with technology but without 'What if not' strategy, without technology but with 'What if not' strategy, and without technology and without 'What if not' strategy. Here flexibility is associated with the number of questions' types on a problem, or with the number of implemented posing strategies (Leikin, Koichu, & Berman, 2009).

METHOD

The participants in this research were pre-service teachers who specialized in mathematics teaching. The research was conducted in the academic year 2013-2014, where the participants were divided into four groups randomly. The groups differed in their use of technology and their use of "What if not?" strategy. Each group included 19-21 participants. The participants in each group were given one hour to carry out the task individually. Moreover, the participants in the two groups who worked with technology used an applet called "the paper pool applet".[1]

The task was adapted from the Silver et al.'s (1996). It was chosen for it fits working with using traditional means as well as technological means. Following is the task text.

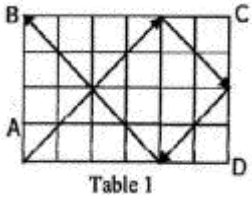
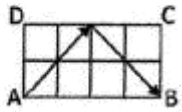
> Imagine billiard tables like the one shown below. Suppose a ball is shot at 45o angle from the lower left corner A of the table. When the ball hits aside of the table, it becomes off at a 45o angle. In Table 1, the ball travels on a 4x6 table and ends up in pocket B, after 3 hits on the sides. In Table 2, the ball travels on a 2x4 table and ends up in pocket B, after 1 hit on the side. In each of the figures shown below, the ball hits the sides several times and then eventually lands on a corner pocket.
>
> 
>
> Table 1
>
> 
>
> Table 2
>
> Based on this situation, pose and write down as many interesting mathematical problems as you can.

Figure 1: The paper pool Task

To analyze the data, we first excluded problems that were not mathematical or unsolvable. For example, the problem "what happens if we change the table's color?" was not considered mathematical.

We considered two types of flexibility for a participant: the number of different problem posed by a participant, and the number of posing strategies that she/he used. This analysis method follows Kontorovich et al. (2011) who analyzed secondary school students' creativity in problem posing.

FINDINGS

Examining the different strategies that the participants used, we found three approaches. The first relates to the object of the problem: a mathematical object or a mathematical relation. The second relates to the givens of the problem: keeping the givens, or varying the givens. The third approach relates to generality of the problem: asking about a specific object, or asking about a general object. We present examples on the different types of problems in the three approaches in Table 2 and Table 3 below. Table 2 presents examples from problems posed on specific object/relation, while table 3 presents problems posed on general objects/relation. Table 2 differentiates between 'keeping the givens' and 'varying the givens', while Table 3 deals only with 'varying the givens', for generality, by its nature, needs varying the givens of a problem.

[1] At http://illuminations.nctm.org/Activity.aspx?id=4219

Table 2: Problems posed on specific mathematical object/relation

Topic \ Givens	Keeping the givens	Varying the givens
Mathematical object	What is the length of the path resulting from a ball hitting the sides of the 2x4 table?	What is the angle that we should use in throwing the ball in order to get equal number of hits in the two given tables?
Mathematical relation	How do the dimensions of the table 2x4 determine the number of ball hits?	What happens to the relation between the number of ball hits and table dimensions on a table with 6 pockets?

Table 3: Problems posed on general mathematical object/relation

Topic \ Givens	Varying the givens
Mathematical object	What are the table's dimensions that guarantee no intersections of the ball path until it hits a pocket?
Mathematical relation	What is the relationship between the table's dimensions and the number of ball hits?

The four groups of pre-service teachers differed in their use of strategies in problem posing. Both of the groups who did not use technology used mostly two strategies: keeping the givens or varying the givens, while asking about a specific mathematical object. The group who used technology but not 'What if not' strategy used mostly four strategies: varying the givens and asking about general mathematical object or relation, while asking about specific mathematical object or relation. The fourth group, who used technology and 'What if not' strategy, used all the six strategies.

As to the different problems posed by a participant, we found ten types of problems: problems about the pocket in which the ball falls, problems about the table's dimensions, problems about the ball's path, problems about the number of the ball hits, problems about the number of thrown balls, problem about the number of pockets, problems about the angle of throwing the ball, problems about the ball speed, geometric problem related to the ball path, asking about two or more of the previous issues.

In addition, only participants who used technology and 'What if not' strategy raised all types of problems. Moreover, only participants who worked with technology posed problems about the speed of the ball, while only participants who worked with 'What if not' strategy posed problems about the number of balls.

DISCUSSION

It was the goal of this research to examine mathematics pre-service teachers' flexibility in problem posing in four environments differing in the use of technology and 'What if not' strategy. The research findings show that the group, who combined between technology and 'What if not'

strategy, used three approaches in problem posing, where these approaches included six strategies, and raised ten problem types. The other three groups of pre-service teachers' used only part of the strategies and types. This implies that combining between these two tools makes mathematical problem posers aware of more posing strategies and problem types. Here the 'What if not' strategy could support the mathematics learners in their varying of task givens (Brown, & Walter, 1990), while technology could support them in their generalizing (Tabach, 2011). The numbers of strategies and problem types used by the group that combined between technology and 'What if not' strategy are greater than those reported in Kontorovich et al. (2011) and Silver et al. (1996). This could be due to the potentialities of the two tools that directed the participating mathematics learners in their problem posing activity.

Moreover, the difference between the flexibility of the participants from the four groups occurred as a result of the different context characteristics. This is present in the findings that only students who used technology asked about the speed of the ball, which could be due to the presence of this factor in the applet interface. On the other hand, the number of balls is not present in the applet interface, so students who worked with technology and without 'What if not' strategy did not pose problems about this variable, while students who worked with 'What if not' did. So, talk conditions are of great implications regarding the type of activity with which mathematics learners are engaged.

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