Using dynamic and interactive technological tools to support conceptual learning of equations among low-achieving students

Osama Swidan[1,2] and **Wajeeh Daher**[2,3] and **Nael Darawsha**[2]

[1]Kaye Academic College, Beer Shiva, Israel; osamasw@gmail.com

[2] Al-Qasemi Academic College, Baka Al-Garbia, Israel; naal1975@gmail.com

[3] An-Najah National University, Nablus, Palestine; wajeehdaher@windowslive.com

Mathematics education researchers have been interested in students' understanding of the equality as equivalence relations. Doing so, they pointed out that the notion of equality is difficult for students to perceive. We provided one pair of 16-year-old low-achieving students with a productive environment (technological tool, supportive teacher and an authentic activity) to support their learning of equality sentences as equivalence relations. We examined the pair of students' routines in this environment. The research results indicated that the students followed a sequence of routines where the teacher and the technology had an effective role. Moreover, students' substantiation routines relied on empirical argument that utilized concrete realizations afforded by the applet.

Keywords: Commognition, Dynamic technology, Low-achieving students, Equation, Equivalence.

Introduction

The mathematics education of low-achieving students has attracted educators' attention for a long time. To support the mathematics learning of these students, one of the recommendations is to conduct a classroom environment that is conducive to learning (Leone, Wilson, & Mulcahy, 2010). This can be done, among other things, by giving students authentic tasks and dynamic tools (National Council for Curriculum and Assessment, 2003; National Council of Teachers of Mathematics, 2000), and, at the same time, by maintaining effective teaching (Ball, 2003), for example through questions. By authentic tasks we mean, tasks that are situated in meaningful contexts that reflect the way tasks might be found and approached in real life. In the present research, we tried to follow these principles by giving low-achieving students authentic activities related to equivalence relations. At the same time, the students worked with an applet suited for learning equations as equivalence relations; issues that have been indicated as critical to algebra (e.g., Stacey & Chick, 2004).

Students' understanding of the equivalence relations

Mathematics education researchers have been interested in students' understanding of the equivalence relations (e.g., Kieran, 1981, 1992; Knuth, Alibali, Hattikudur, McNeil, & Stephens, 2008). Knuth et al. (2008) argue that the notion of equality is often complex, and thus difficult for students to perceive. Furthermore, Kieran (1992) considered the equivalence relations as a pre-requirement for understanding structural representations such as equations.

Knuth et al. (2008) examined middle school (grades 6-8) students' definition of the equal sign. They found that those students had three types of conceptions: a relational conception (when the student expressed the idea that the equal sign represented an equivalence relation between two quantities), operational conception (when the student expressed the idea that the equal sign meant "add the numbers" or "the answer"), and other conceptions; for example, when the student used the word "equal" in the definition. Several researchers expressed the view that helping students acquire a relational conception of the equality sign would help them succeed in algebra and beyond (e.g., Hunter, 2007; Knuth et al., 2008). Generally, this concept, together with the related concepts, as equivalence and equation, are complex ones and difficult for students to understand (Hunter, 2007; Kieran, 1981).

A productive environment for students' learning of the equivalence relations

Students' difficulties in understanding the equivalence relation could be lessened in a learning environment that includes authentic tasks (Taylor-Cox, 2003), technology (Jones & Pratt, 2006) and teacher's guidance. As to the use of technology to assist the learning of the equivalence relations, Jones and Pratt (2006) report an experiment in which two students connected an onscreen '=' object with other arithmetic objects, which supported them in developing relational conceptions of the equal sign. As to the use of authentic activities to assist the learning of the equivalence relation, Taylor-Cox (2003) describes the Pan Balance scales as a tool to demonstrate equality, where students need to use and make scales. As to the teacher's guidance as means to facilitate students' learning of the equivalence relation, researchers have indicated the importance of the teachers' role and guidance in learning mathematics in general (NCTM, 2000), and learning the equivalence relation in particular (e.g., Taylor-Cox, 2003). Taylor-Cox (2003) describes the mathematics teacher's role in enhancing students' learning, for example by asking questions that promote mathematical dialogue and understanding. The mathematics teacher’s actions are part of the classroom routines (using Sfard's terms) that assist the students in their mathematics learning.

We designed the learning environment taking into consideration the role of technology, the role of the teacher, and the type of the tasks. To better understand the students' learning in this environment, we analyzed this learning using Sfard’s commognitive approach. Especially, we concentrated on the evolution of routines’ use. In the following section, we briefly outline the commognitive approach.

Routines in the mathematics classroom

Sfard (2008) presents four components of the mathematical discourse that help analyze it: words, visual mediators, narratives and routines. *Mathematical words* are used by the participants in a mathematical discourse to express and communicate with the other participants about mathematical ideas. In this discourse, a student learns new uses of previously encountered mathematical words, but may also learn new mathematical words. *Visual mediators* are visual objects and means with which participants of mathematical discourses identify mathematical ideas. They include symbols such as numerals, algebraic letters, tables, graphs and diagrams. A *narrative* is a spoken or written text that describes objects, or relations between objects or activities with or by objects, and that could be accepted or rejected within the mathematical discourse. Mathematical examples of narratives could be theorems, definitions and equations.

Sfard (2008) defined *Routines* as “repetitive patterns characteristic of the given discourse” (p. 134). They characterize the use of mathematical words and visual mediators or the creation, substantiation or change of mathematical narratives. Examples on typical mathematical routines are methods of calculations and of proof (Sfard, 2008). She divides routines into explorations that aim to further discourse through producing or verifying endorsable narratives (as verifying a mathematical conjecture or proving a mathematical relation); deeds that aim to change the actual objects, physical or discursive, not just the narratives; and rituals that aim to create and sustain social approval with other participants in the mathematical discourse. Furthermore, rituals could involve imitations of other participants' routines (Berger, 2013). Sfard further divided explorations into three types: construction that aims to create new endorsable narratives, substantiation that aims to decide whether to endorse previously created narratives, and recall that aim to summon narratives endorsed in the past.

Previous research has used the commognitive framework in different ways to examine the four components of the mathematical discourse, or just some of them (e.g., Berger, 2013; Viirman, 2012). Little research has been done on students' routines while learning the equality sentences as equivalence relations, where most of the research was done on students' word use or narratives related to these concepts. The present research intends to study the routines of low-achieving students while learning equations as an equivalence relations between quantities. The main research question is: what are the characteristics of low-achieving students' routines in the course of learning equations as an equivalence relations between quantities in a productive learning environment?

The design of the study

To answer the research question we analyzed approximately three hours of learning by Noha and Maha, one pair1 of 16-year-old low-achieving students in the math class taught by the third author.

The experiment took place in a school of low-achieving students who want to graduate as car mechanics or house/car electricians. The students volunteered to participate in four after-school meetings that aimed to teach the equations as an equivalence relations. In this study, we concentrated on the third meeting, which dealt with learning the equivalence between the two sides of an equation when performing arithmetic operation. The students who participated in this study had prior knowledge in operator precedence and the substitution of numeric values in algebraic expressions. They were not familiar with using technological software in learning mathematics. The two students shared a single computer, and the third author briefly introduced them to the functions of the software.

The students were video-recorded and their computer screens were captured. The video recording was performed with a computer program that captured the footage in two different windows; one for the computer screen and the other for the student's body. The third author conducted the learning activity. His main role was to ask clarification questions. The pair of students carried out four tasks presented in Figure 1.

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[1] For reasons of space, we decided to perform the micro-analysis of the learning process with one pair of students from the three pairs participating in the research project.

Task 1

- Enter the expression 6x in the red pan and 18 in the blue. What happened to the pans? Why?
- Change the slider until the pans have equal values. Why do the pans have equal value?
- Add the value 2 to the red pan. What happed and why? What should you do now to make the pans balanced?
- Subtract the value 2 from the red pan. What happed and why? What should you do now to make the pans balanced?

Figure 1. Example of a task given to the students

The technological tool used in the experiment

The technological tool used in our study is the interactive applet Pan Balance Expressions (PBE; NCTM, 2015; Fig. 2). The interactive applet PBE allows numeric or algebraic expressions to be entered and compared. Students can "weigh" the expressions they want to compare by entering them on either side of the balance. Using this interactive applet, students can investigate the equivalence of equation. PBE consists of four main windows: a) the slider window, which allows the student to vary the x- values; b) the pans window, which contains symbolic expressions entered by the users; c) the keyboard window, which enables the student to enter and edit expressions in the pans; d) the graphic window, which represents the graphs of the expressions entered in the pans.

Data analysis

To analyze the data, we categorized the routines, as suggested in Sfard (2008). We considered a routine to be an exploration when the student's goal, from performing the routine, was to arrive at a narrative. More specifically, we considered a routine to be an exploration of the type 'construction', when its goal was to arrive at a mathematical relationship. Moreover, we considered a routine to be an exploration of the type 'construction', its goal was to verify a relationship that was arrived at or conjectured. Other categories that we found are: teacher's request (when the teacher requested the students to do an action), and students' actions with the applet (when the students worked with the applet for different reasons).

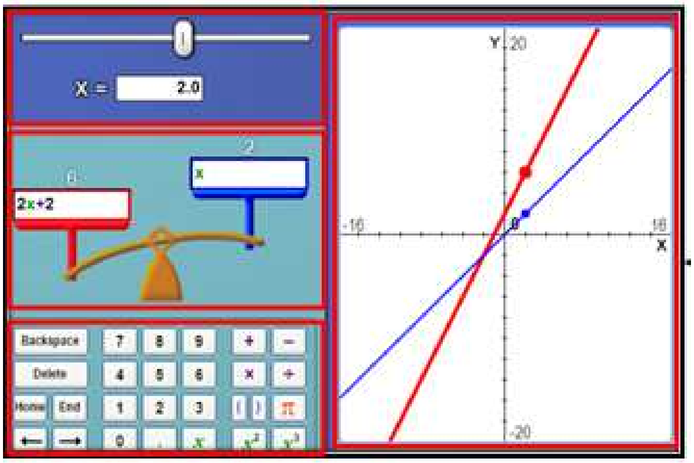

Figure. 2: The interface of the Pan Balance applet

Results

The pair of low-achieving students worked with three groups of narratives; (a) Solving the equation Ax=B using the applet; (b) constructing the equivalence equations resulting from performing the same allowed operation on both sides of the equation; (c) solving linear equations using the equivalence principle. In the present paper, we will present students' routines related to constructing the equivalence concept resulting from performing the same allowed operation on both sides of the equation.

Transcript 1 describes the pair of students' work while adding the same number to both sides of an equation. At this phase, the expression 6x was in the red pan and 18 in the blue one. The slider was at x=3, which mean the pans were in balance.

25	T:	Add the number two to the blue pan
26	N:	*(she added two to the blue pan causing the red pan to rise)*
27	T:	What did you see?
28	N:	Eighteen plus two
29	T:	What happened?
30	M:	It rose.
31	N:	It is not equal; the red pan rose and the blue fell.
32	T:	Why did this occur?
33	M:	(*Looking at the Pan Balance*) Because we added the number two to the blue pan. They are not balanced; one pan is higher than the other.
34	T:	Could they balance now?
35	M:	*(adds 2 to the red pan)*
36	M:	Yes, if we added the number two to the red pan.
37	N:	Yes they are balanced now.
38	T:	Why are they balanced now?
39	N:	Previously there were 18 on both sides. Thereafter, we added two to the blue pan. It totaled 20. Now I added two to the 6x and it also totaled 20. It is now equal.

Transcript 1: Adding the same number to the two sides of an equation

This transcript illustrates the pair of students' routines, which led to the endorsement of the narrative "*Yes, if we added the number two to the red pan*" [36]. Students' routines started with a teacher's request [26] with an overall intention to allow the student to construct a narrative related to adding a number to an equation. The students got engaged in actions with the applet [26]. The teacher then started a construction routine, with the intention to make the students aware of the effect of adding a number on one pan [27-31]. Then the teacher started a routine of substantiation [32-33]. It can be seen that the students' exploration constituted of the following sequence of routines: teacher's request, students' actions with the applet, students' construction of a narrative, teacher's questioning, and students' substantiation of the narrative. The pair of students performed again the same sequence of routines to explore how to make the two pans equal: teacher's request [34], students' actions with the applet [35], students' construction of a narrative [36-37], teacher's question [38] and students' substantiation of the narrative [39].

In their exploration of the narrative related to subtracting a number from the two sides of an equation, the pair of students needed just one sequence of routines. Moreover, in their exploration of the narrative related to multiplying the two sides of an equation by the same number, the pair of students skipped performing actions with the applet to construct the narrative. However, and as transcript 2 shows, they performed these actions with the applet to substantiate the narrative about the equivalence of an equation under multiplication.

86	T:	What would happen if you multiplied the expressions in the pans by the same number?
87	N:	When multiplying, the balance of the two pans would remain unchanged.
88	N:	[*she inserted the expression 6x on one pan and 18 on the other; thereafter she fixed x=3 to balance the pans*].
89	N:	I will multiply both sides by 2.
90	N:	[*She multiplied both sides by 2*].
91	N:	I got it right.

Transcript 2: Multiplying the two sides of an equation by the same number

This transcript illustrates a modified sequence of routines: teacher’s request [86], conjecture (as a part of a construction) [87], actions with the applet [88], substantiation [89-91].

The data analysis revealed some characteristics of students' routines. First, routines started with a teacher’s request or questioning. It seems that one of the teacher’s routines in the low-achieving classroom was to start the learning process by requesting the students to act or to answer. Second, the pair of students followed a sequence of routines to arrive at each of the narratives. This sequence consisted of teacher’s request, students’ actions with the applet, constructing a narrative, and substantiating it. This sequence of routines was not kept as is for every narrative, but a variation of it was followed. Third, students’ actions with the applet, what we could call *deed routines*, supported the low- achieving students in their exploration routines, whether they were constructions or substantiations. Fourth, the data analysis revealed a pattern of evolution of the routines associated with the successive narratives, where the number of routines needed for the students to endorse narratives was decreased for each group of narratives.

Discussion

The goal of the present research was to examine the routines of a pair of low-achieving students, while learning the equivalence relations in a productive learning environment. The students worked with the Pan Balance, which illustrate the equation concept. Working with it, they actually worked with visual mediator which signifying the mathematical objects and relations (Sfard, 2008, p. 224). Moreover, the students' routines regard using the visual mediator were visual and dynamic, where they could scan the Pan Balance and manipulate it, and consequently watch the effects of this manipulation on the equivalence relations. It could be claimed that these visual and dynamic routines helped the low-achieving pair of students to signify the equivalence relation through construction and substantiation routines. Furthermore, the applet constituted for the pair of low-achieving students a prompt for construction and substantiation routines.

It was observed that the pair of low-achieving students used a sequence of routines: teacher's request, students' actions with the applet, students' construction of a narrative, teacher's questioning and students' substantiation of the narrative. Moreover, students' use of the sequence of routines satisfied the variability and flexibility principles (Felton & Nathan, 2009; Sfard, 2008, pp. 202-205), i.e. the students varied their use of the sequence to meet their needs. This happened for example, when they engaged with multiplying the two sides of an equation by the same number. Constructing the appropriate equivalence narrative, they skipped performing actions with the applet, but performed these actions to substantiate the narrative.

The sequence of routines described above shows the effect of the teacher's routines and of technology affordances on students' routines. It seems that the teacher's initiation of students' construction and substantiation routines was a prompt for them to follow routines that supported their successful construction of equivalence narratives. As for the technology affordances, the Pan Balance applet allowed the pair of low-achieving students to perform actions that supported them in their construction and substantiation of the equivalence narratives (e.g., scanning the equilibrium of the Pan Balance). Moreover, we argue regarding the pair of students' substantiation routines, that they relied on empirical argument that utilized the "concrete realizations of the focal signifiers and relies on their perceptually accessible features" (Sfard, 2008, p.233). This type of substantiation is probably expected of low-achieving students.

The present research reports the routines of one pair of low-achieving students. It shows that a productive learning environment that combines teacher's initiation and questioning, technology and authentic tasks will support these students' routines for arriving at mathematical narratives. Research that engages more low-achieving students' is needed to confirm this research findings regarding their routines in similar environments.

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