# Solving Word Problems and Working with Parameters in the spreadsheets environment

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**Abstract**

*Students have difficulty working with the parameter concept when solving mathematical word problems and when working in mathematical contexts. Technological tools can be used to overcome such difficulties and one type of the technological tools which has been recently used in the mathematics classroom is the spreadsheets. This article describes how preservice teachers experienced solving mathematical word problems using the spreadsheets and how they developed their notions of algebraic concepts, especially the parameter concept, as a result of this use. The findings indicate that the preservice teachers developed their utilization of the spreadsheet environment to solve word problems. At the same time, they decreased the difficulties they confronted when working with parameters (before using the spreadsheets) and developed their notion of this concept (as a result of working with the spreadsheets), but they did not grasp the concept fully.*

## 1. Introduction

Researchers point at the advantages of using computers in education in general ([5], [15], [16], [17]) and in mathematics education in particular ([9], [22], [24]). The NCTM [14] states its position on using technology in mathematics education, pointing at technology as an essential tool for learning mathematics in the current century, and advising that schools should ensure that their students have access to technology. The NCTM [14] mentions the spreadsheets as one of the technology tools which students can use to support them reason mathematically, make sense of mathematical ideas, and gain access to mathematical content and problem-solving contexts. This article describes how preservice teachers utilized the spreadsheets environment to make sense of algebraic concepts and solve algebraic word problems, especially how they made sense and gained access to the ideas of variables and parameters.

## 2. Literature review

The literature review describe researches that have relations with issues that can be related to the research topic, i.e. solving word problems and being aware of the characteristics of variables and parameters, and distinguishing between them.

### 2.1 Problem solving in the spreadsheets environment:

Dettori et al [7] describe how students can learn the difference between variables and parameters when solving word problems using the spreadsheets.

Sutherland and Rojano [20] examined how students aged 10-11 years solved algebra problems in the spreadsheets environment. Doing so, they also looked at the development of students' use of symbolic language. Working with the spreadsheets the students represented the unknown with a cell in a spreadsheet, expressed the mathematical relation in the problem with a relation between two cells, and changed the value of the cells to obtain the solution of the problem. Thus, the spreadsheets environment provided the 10-11 years students with the opportunity to work with the unknown, the variable and with mathematical relations. The research showed that most of the 10-11 years students did not think spontaneously in terms of general formulae when they met the spreadsheets environment for the first time. At the beginning, the students' thinking was in terms of the example, but gradually, most of the students could work with general objects when needed. The students transited from a local numeric concept of cause-result to the concept of a relation which could be represented by a rule with the spreadsheets language; a language that could easily be translated into the language of algebra.

Ainley [3] studied how the context of physical experiment and spreadsheets environment that utilizes the graphing potential of the spreadsheets can introduce 11 years old students to the power of generalizing through formal algebraic notation. This was done while the students were asked to solve word problems.

Abramovich and Nabors ([1], [2]) studied how middle school students solved word problems in the spreadsheets environment. Rojano [18] studied how 15-16 years old students who had difficulties in mathematics solved word problems in the spreadsheets environment.

Daher [6] described how preservice teachers utilized the spreadsheets to solve word problems, and points that spreadsheets provided an environment which encourages multiple representations of mathematical objects and concepts.

Following is the description of some researches which were involved with introducing variables to students from different ages.

**2.2 The transition from arithmetic to algebra**:

Researches in the last decades have pointed at symbolic or/and cognitive transformations which mark the difference between algebraic and arithmetic thinking, where some of the transformations are related to different interpretations of letters, the equality concept, and the accepted graphical or symbolic conventions ([8], [20]). Felloy and Rojano [8] said that the transformations which the student needs to make in order to absorb the algebraic language could be looked at as breaking points that separate the two types of thinking. They also emphasized that these breaking points in the individual plane represent historic breaking points in the arithmetic thinking. These breaking points make it possible for the algebraic thinking to appear. This transformation from the arithmetic thinking to the algebraic thinking is not accomplished with difficulties.

Lee [13] described the algebra as a small culture in a bigger culture which is mathematics. Lee [13] described the transition of the student from arithmetic to algebra as a cultural shock.

To overcome the difficulties of students in transiting from arithmetic to algebra researchers and educators have used computer environments and programs. Examples of such environments and programs are the spreadsheets, the applets, the middles, etc.

**2.3 Working in a computer environment to make an easy transition to algebra**:

Some researchers suggested the computer environment as an environment which could assist students in transiting to algebra concepts and procedures. Sutherland [19] described a Logo environment where students aged approximately 10 were introduced to the variable concept. She reported that the connections made by the participants between the variable concept in Logo and the variable concept in algebra were influenced by the nature and extent of experience in the Logo environment. Graham and Thomas [12] described how they used the graphic calculator to teach algebra to students aged 13-14 years. Their findings indicated that by using the graphic calculator, students can absorb better the concept of a letter as a specific unknown or general number. Tall and Thomas [21] reported evidences that learning environments which use the computer assist the students' understanding of the variable concept more than one that does not use the computer. Tall and Thomas' experience was done in an environment that combined programming in BASIC, physical activities which simulate computer storage and manipulation of variables, and specific software which evaluated expressions in standard mathematical notation. The software was designed to enable the user to explore examples and non-examples of a concept, in this case equivalent and non-equivalent expressions. Tall and Thomas said that such an approach significantly improved the understanding of higher order concepts in algebra.

The previous researchers used the computer environment to help students in their transition from arithmetic to algebra, while other researchers constructed a computer environment which helped learn a total algebraic topic. For example Yerushalmy, Katriel & Sternberg [25] designed an e-book which is constituted of a visual environment that includes several applets with which the student can learn the function topic. Yerushalmy and Schwartz [26] argued that a visual environment opens new possibilities for the students to understand algebraic concepts.

One of the computer programs which students can have access to more than other programs is the spreadsheets, where its excel version comes with the Microsoft Office which is probably found in every school. This gives the spreadsheets priority over other computer programs as a tool for learning mathematics. Some researchers paid attention to the potentials of the spreadsheets in learning mathematics and used it as a tool for teaching and learning mathematics. Below I describe some of these researches.

Haspekian [10] said that the spreadsheets offer access to the meaning of algebra through the use of formulae and graphing and specifically to the meaning of variable through the notion of a ‘variable cell’ and a ‘variable column’. Wilson, Ainley and Bills [33] analyzed the mediation of the variable cell and the process of naming a column under the guidance of the teacher. They found that "the dynamic metaphors of *change* and *dragging* together with the process of *naming* appeared to support the evolution of meaning for variable” (p. 327).
Dettori et al. [7] described the advantages and disadvantages of using the spreadsheets in algebra. The following points were mentioned as influencing negatively the use of spreadsheets in algebra:

- The student solves the mathematical problem without being aware of the equation involved in the solution. This situation results because the equal sign in the spreadsheets is actually assigning a computed value to the cell, whereas the equal sign in algebra represents a relation. This difference influences the solution methods in algebra and in the spreadsheets: solving in algebra involves manipulations of letters, whereas solving in the spreadsheets involves continuous approximations till arriving at the solution.
- The unknown is absent in the spreadsheets solution. This situation results because the formulae in the spreadsheets are not equations but functions, so the cell name represents at most a functional variable and not algebraic unknown.
- Transiting from the spreadsheets solution to the algebraic solution requires making a synthesis from the partial relation that exists in the spreadsheets environment to one or more algebraic equations that represent the mathematical problem. This synthesis is difficult to perform without the interference of the teacher. This may be the case because the possibility to arrive at the solution through continuous approximations may discourage the student to do the required synthesis.
- It is difficult to prove the uniqueness of the solution arrived at in the spreadsheets environment, whereas this is an easy task when the solution is algebraic.
- When solving a problem that has more than one solution in the spreadsheets environment without algebraic considerations, it is difficult to distinguish the structural difference between this sort of problem and a problem which has one solution. In addition, it is

difficult to find the set of all solutions in the spreadsheets environment, whereas it is easy to do that with algebraic considerations.

The following points are mentioned by Dettori et al. as advantages resulting from using the spreadsheets in algebra:

- Learning in the spreadsheets environment, students can distinguish which elements are involved in a problem and to express part of the relations in the problem. Doing so, they get used to the analysis needed for approaching the problem algebraically.
- Learning in the spreadsheets environment, students can be aware of the cell name as a functional variable, but this can be done with the help of the teacher who helps them arrive at the abstraction stage. The teacher can do that by pointing at the analogies and differences between the spreadsheets environment and the algebraic one.
- Using the spreadsheets as an environment for solving word problems enables the teacher to direct the students to think about the range and contexts of the possible solutions to a problem.
- The spreadsheets environment can be utilized as a tool for a successful introduction to the functions topic. This is so due to the easiness with which we can build and change tables, so this property facilitates making conjectures regarding patterns that functions can have, and facilitates testing those conjectures.
- The spreadsheets environment can be utilized as a tool for the student to understand the concept of generalizing a problem and to distinguish between variables and parameters.

Dettori et al. [7] emphasize that most of the disadvantages of the spreadsheets could be overcome with the help of the teacher who can direct the students to the similarities and differences between the mathematics done in the spreadsheets and the one done with algebra.
Depending on the description above, It can be claimed that the spreadsheets tool is appropriate for introducing the difference between variables and parameters. This research wants to examine this claim when preservice teachers are involved.

## 3. The research rationale:

"Making sense of letters is one of the fundamental problems in the learning of algebra." ([4], p. 129). This could be a problem not only for school students but also for preservice teachers who major in mathematics. So verifying the concepts of those preservice teachers of variables, unknowns and parameters would help teachers' educators prepare appropriate activities for preservice teachers who study mathematics, so that they overcome their difficulties distinguishing among these entities. The claim of Dettori et al. [7] that the spreadsheets could be an appropriate environment for learning about variables and parameters encouraged me to use the spreadsheets as a mathematical environment in which the preservice teachers solve word problems, and doing so, they gradually recognize the properties of variables and parameters and distinguish between them.

## 4. Research questions:

1. How would preservice teachers majoring in mathematics utilize the spreadsheets potential to solve word problems?
2. How would preservice teachers majoring in mathematics use the spreadsheets potential to represent parameters in order to solve word problems, where this use shortens the solution procedure?
3. How would the spreadsheets environment help preservice teachers who study mathematics to grasp the parameter concept and to distinguish between the parameter and the variable concepts?

## 5. The research methodology:

### 5.1 Research setting:

The experiments took place in two classes of second year preservice teachers who studied mathematics and computers at Al-Qasemi Academic College of Education. The observations were held during the academic years 2006-2007 and 2007-2008. The number of preservice teachers participating in the research was twenty seven in the academic year 2006-2007 and thirty four in the academic year 2007-2008. These preservice teachers had already taken a course called “Spreadsheets Integration in Education”. In that course, the preservice teachers were introduced first to the various functions of the spreadsheets as an environment in which various computations can take place, and then they were introduced to the spreadsheets as an educational environment which teachers can use as an alternative or additional method in the classroom, especially in the mathematics classroom.

In this experiment, I wanted to (1) examine how the preservice teachers utilize the spreadsheets to solve mathematical word problems, and specifically how they use the parameter option in the spreadsheets (fixing the value of a cell) to solve mathematical word problems, (2) clarify the preservice teachers’ concepts of the parameter. At the beginning the preservice teachers were required to write at least three examples on parameters and then write the definition of the parameter. Afterwards, the preservice teachers were given four word problems taken from the literature. The word problems were modified, so that an additional part was added. The preservice teachers were required in that part to find one method or frame through which all the parts of a word problem could be solved together. This assignment was given while discussing the role of spreadsheets in mathematics education in the course “didactics of mathematics teaching” which the author of this article taught. The preservice teachers worked with the word problems for three weeks, i.e. four and a half hours in overall. After solving each word problem a discussion was held regarding the best spreadsheets method to solve that problem. After they finished solving the four word problems in the spreadsheets environment, the preservice teachers were again required to write at least three examples on parameters and then write the definition of the parameter.

### 5.2 The word problems:

The word problems were taken from the literature and are described in appendix 1.

### 5.3 Data collecting tools:

Questionnaire:

The questionnaire included the following two items:

a) Give three examples on parameters.
b) State the definition of a parameter.

Observations:

I observed the work of the preservice teachers when they solved word problems in the spreadsheets environment, and kept record of the work of one of them, let us call him here Salim.

### 5.4 Data analysis tools:

In this research I use content analysis to categorize the preservice teachers' conceptions of the parameter and represent the percentages of preservice teachers who had each parameter concept. The content analysis method is described by Holsti [11] as "any technique for making inferences by objectively and systematically identifying specified characteristics of messages" (p. 14). In this research the characteristics of the preservice teachers’ conceptions of parameter were coded by two coders, and the computed Cohen's Kappa of the reliability of the coding was found to be 85%. In addition to categorizing the preservice teachers' conceptions of parameters, the development of one preservice teacher's conception of parameters will be followed along his work in the spreadsheets environment.

## 6. Findings:

### 6.1 Utilizing the potentials in the spreadsheets environment to solve word problems:

The utilization of the spreadsheets environment to solve word problems will be presented by describing the way Salim (a pseudonym) utilized the spreadsheets to solve the word problems. The presentation will also take care of how Salim's solution method developed from one word problem to another. This development is similar to the development of the other preservice teachers' methods. In 'The Measurement of a Field' problem, Salim solved the first three parts by building columns for the length, the width and the circumference of the field. Figure 1 shows this method.

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | width | length | circumference | | width | length | circumference | |
| 2 | 5 | 10 | 30 | | 5 | 15 | 40 | |
| 3 | 6 | 12 | 36 | | 6 | 18 | 48 | |
| 4 | 7 | 14 | 42 | | 7 | 21 | 56 | |
| 5 | 8 | 16 | 48 | | 8 | 24 | 64 | |
| 6 | 9 | 18 | 54 | | 9 | 27 | 72 | |
| 7 | 10 | 20 | 60 | | 10 | 30 | 80 | |
| 8 | 11 | 22 | 66 | | 11 | 33 | 88 | |
| 9 | 12 | 24 | 72 | | 12 | 36 | 96 | |
| 10 | 13 | 26 | 78 | | 13 | 39 | 104 | |
| 11 | 14 | 28 | 84 | | 14 | 42 | 112 | |
| 12 | 15 | 30 | 90 | | 15 | 45 | 120 | |
| 13 | 16 | 32 | 96 | | 16 | 48 | 128 | |
| 14 | 17 | 34 | 102 | | 17 | 51 | 136 | |
| 15 | 18 | 36 | 108 | | 18 | 54 | 144 | |
| 16 | 19 | 38 | 114 | | 19 | 57 | 152 | |
| 17 | 20 | 40 | 120 | | 20 | 60 | 160 | |
| 18 | 21 | 42 | 126 | | 21 | 63 | 168 | |
| 19 | 22 | 44 | 132 | | 22 | 66 | 176 | |
| 20 | 23 | 46 | 138 | | 23 | 69 | 184 | |
| 21 | 24 | 48 | 144 | | 24 | 72 | 192 | |
| 22 | 25 | 50 | 150 | | 25 | 75 | 200 | |
| 23 | 26 | 52 | 156 | | 26 | 78 | 208 | |
| 24 | 27 | 54 | 162 | | 27 | 81 | 216 | |
| 25 | 28 | 56 | 168 | | 28 | 84 | 224 | |
| 26 | | | | | | | | |

**Figure 1: Solving the field measurement by dragging cells to get columns of constant difference between consecutive cells in the first column**

Salim solved the first part of the 'field measurement problem' by building columns for the length, width and circumference of the field. This was done by taking into account the relations between the unknowns or/and the given, where each column was built by dragging the first cell downwards. The first column was built by making constant the difference between any two consecutive cells in it. After building all the columns by dragging, Salim looked at the perimeter column to find the cell

containing the given perimeter. Finding this cell, Salim looked at the cells of the width and length beside it, thus he found the unknowns required in the word problem.
To solve the second part of the problem he used the same configuration of columns, but to solve the third part he built additional columns. Figure 2 shows the equations of Salim's method.

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | width | length | circumference | | width | length | circumference |
| 2 | 5 | =2*A2 | =2*(A2+B2) | | 5 | =3*E2 | =2*(E2+F2) |
| 3 | 6 | =2*A3 | =2*(A3+B3) | | 6 | =3*E3 | =2*(E3+F3) |
| 4 | 7 | =2*A4 | =2*(A4+B4) | | 7 | =3*E4 | =2*(E4+F4) |
| 5 | 8 | =2*A5 | =2*(A5+B5) | | 8 | =3*E5 | =2*(E5+F5) |
| 6 | 9 | =2*A6 | =2*(A6+B6) | | 9 | =3*E6 | =2*(E6+F6) |
| 7 | 10 | =2*A7 | =2*(A7+B7) | | 10 | =3*E7 | =2*(E7+F7) |
| 8 | 11 | =2*A8 | =2*(A8+B8) | | 11 | =3*E8 | =2*(E8+F8) |
| 9 | 12 | =2*A9 | =2*(A9+B9) | | 12 | =3*E9 | =2*(E9+F9) |
| 10 | 13 | =2*A10 | =2*(A10+B10) | | 13 | =3*E10 | =2*(E10+F10) |
| 11 | 14 | =2*A11 | =2*(A11+B11) | | 14 | =3*E11 | =2*(E11+F11) |
| 12 | 15 | =2*A12 | =2*(A12+B12) | | 15 | =3*E12 | =2*(E12+F12) |
| 13 | 16 | =2*A13 | =2*(A13+B13) | | 16 | =3*E13 | =2*(E13+F13) |
| 14 | 17 | =2*A14 | =2*(A14+B14) | | 17 | =3*E14 | =2*(E14+F14) |
| 15 | 18 | =2*A15 | =2*(A15+B15) | | 18 | =3*E15 | =2*(E15+F15) |
| 16 | 19 | =2*A16 | =2*(A16+B16) | | 19 | =3*E16 | =2*(E16+F16) |
| 17 | 20 | =2*A17 | =2*(A17+B17) | | 20 | =3*E17 | =2*(E17+F17) |
| 18 | 21 | =2*A18 | =2*(A18+B18) | | 21 | =3*E18 | =2*(E18+F18) |
| 19 | 22 | =2*A19 | =2*(A19+B19) | | 22 | =3*E19 | =2*(E19+F19) |
| 20 | 23 | =2*A20 | =2*(A20+B20) | | 23 | =3*E20 | =2*(E20+F20) |
| 21 | 24 | =2*A21 | =2*(A21+B21) | | 24 | =3*E21 | =2*(E21+F21) |
| 22 | 25 | =2*A22 | =2*(A22+B22) | | 25 | =3*E22 | =2*(E22+F22) |
| 23 | 26 | =2*A23 | =2*(A23+B23) | | 26 | =3*E23 | =2*(E23+F23) |
| 24 | 27 | =2*A24 | =2*(A24+B24) | | 27 | =3*E24 | =2*(E24+F24) |
| 25 | 28 | =2*A25 | =2*(A25+B25) | | 28 | =3*E25 | =2*(E25+F25) |
| 26 | | | | | | | |

**Figure 2: Solving the field measurement by dragging to get columns: equation view**

To solve the first and the second parts of the chocolates problem, Salim continued using columns, but this time he used parameters to take care of the changing relation between the mathematical objects. To take care of the big number of the distributed chocolates in part 2, Salim started from 10 to indicate the first group's number of chocolates and proceeded by 10s. Figure 3 and Figure 4 show how Salim solved the first two parts of the problem.

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | group 1 | group 2 | group 3 | total | | | group 2 : group 1 | group 3 - group 2 |
| 2 | 1 | 4 | 14 | 19 | | | 4 | 10 |
| 3 | 2 | 8 | 18 | 28 | | | | |
| 4 | 3 | 12 | 22 | 37 | | | | |
| 5 | 4 | 16 | 26 | 46 | | | | |
| 6 | 5 | 20 | 30 | 55 | | | | |
| 7 | 6 | 24 | 34 | 64 | | | | |
| 8 | 7 | 28 | 38 | 73 | | | | |
| 9 | 8 | 32 | 42 | 82 | | | | |
| 10 | 9 | 36 | 46 | 91 | | | | |
| 11 | 10 | 40 | 50 | 100 | | | | |
| 12 | 11 | 44 | 54 | 109 | | | | |
| 13 | 12 | 48 | 58 | 118 | | | | |
| 14 | 13 | 52 | 62 | 127 | | | | |
| 15 | 14 | 56 | 66 | 136 | | | | |
| 16 | | | | | | | | |
| 17 | | | | | | | | |
| 18 | group 1 | group 2 | group 3 | | | | | |
| 19 | 10 | 40 | 50 | 100 | | | | |
| 20 | 20 | 80 | 90 | 190 | | | | |
| 21 | 30 | 120 | 130 | 280 | | | | |
| 22 | 40 | 160 | 170 | 370 | | | | |
| 23 | 50 | 200 | 210 | 460 | | | | |
| 24 | | | | | | | | |

**Figure 3: Solving the first and second parts of the chocolate problem by dragging and by using parameters**

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | group 1 | group 2 | group 3 | total | | | group 2 : group 1 | group 3 - group 2 |
| 2 | 1 | =$G$2*A2 | =B2+$H$2 | =A2+B2+C2 | | | 4 | 10 |
| 3 | 2 | =$G$2*A3 | =B3+$H$2 | =A3+B3+C3 | | | | |
| 4 | 3 | =$G$2*A4 | =B4+$H$2 | =A4+B4+C4 | | | | |
| 5 | 4 | =$G$2*A5 | =B5+$H$2 | =A5+B5+C5 | | | | |
| 6 | 5 | =$G$2*A6 | =B6+$H$2 | =A6+B6+C6 | | | | |
| 7 | 6 | =$G$2*A7 | =B7+$H$2 | =A7+B7+C7 | | | | |
| 8 | 7 | =$G$2*A8 | =B8+$H$2 | =A8+B8+C8 | | | | |
| 9 | 8 | =$G$2*A9 | =B9+$H$2 | =A9+B9+C9 | | | | |
| 10 | 9 | =$G$2*A10 | =B10+$H$2 | =A10+B10+C10 | | | | |
| 11 | 10 | =$G$2*A11 | =B11+$H$2 | =A11+B11+C11 | | | | |
| 12 | 11 | =$G$2*A12 | =B12+$H$2 | =A12+B12+C12 | | | | |
| 13 | 12 | =$G$2*A13 | =B13+$H$2 | =A13+B13+C13 | | | | |
| 14 | 13 | =$G$2*A14 | =B14+$H$2 | =A14+B14+C14 | | | | |
| 15 | 14 | =$G$2*A15 | =B15+$H$2 | =A15+B15+C15 | | | | |
| 16 | | | | | | | | |
| 17 | | | | | | | | |
| 18 | group 1 | group 2 | group 3 | | | | | |
| 19 | 10 | =$G$2*A19 | =B19+$H$2 | =A19+B19+C19 | | | | |
| 20 | 20 | =$G$2*A20 | =B20+$H$2 | =A20+B20+C20 | | | | |
| 21 | 30 | =$G$2*A21 | =B21+$H$2 | =A21+B21+C21 | | | | |
| 22 | 40 | =$G$2*A22 | =B22+$H$2 | =A22+B22+C22 | | | | |
| 23 | 50 | =$G$2*A23 | =B23+$H$2 | =A23+B23+C23 | | | | |
| 24 | | | | | | | | |

**Figure 4: Solving the first and second parts of the chocolate problem by dragging and by using parameters: equation view**

To solve the third part of the chocolate problem, Salim followed a different strategy: building the spreadsheet configuration row by row while watching the overall number of chocolates. When Salim arrived at the understanding that adding 10 every time, in the first column, would result in a long way to the solution, he started to increase the value of the cell in the first column by bigger numbers, till he got 650 in the overall cell. Figures 5 and 6 describe this method.

| | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | group 1 | group 2 | group 3 | total | | | group 2 : | group 3 - | group 2 | |
| 2 | 10 | 10 | 60 | 80 | | | 1 | 50 | | |
| 3 | 20 | 20 | 70 | 110 | | | | | | |
| 4 | 30 | 30 | 80 | 140 | | | | | | |
| 5 | 40 | 40 | 90 | 170 | | | | | | |
| 6 | 200 | 200 | 250 | 650 | | | | | | |
| 7 | | | | | | | | | | |

**Figure 5: Solving the third part of the chocolate problem without dragging and with parameters**

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | group 1 | group 2 | group 3 | total | | | group 2 : group 1 | group 3 - group 2 |
| 2 | 10 | =$G$2*A2 | =B2+$H$2 | =A2+B2+C2 | | | 1 | 50 |
| 3 | 20 | =$G$2*A3 | =B3+$H$2 | =A3+B3+C3 | | | | |
| 4 | 30 | =$G$2*A4 | =B4+$H$2 | =A4+B4+C4 | | | | |
| 5 | 40 | =$G$2*A5 | =B5+$H$2 | =A5+B5+C5 | | | | |
| 6 | 200 | =$G$2*A6 | =B6+$H$2 | =A6+B6+C6 | | | | |
| 7 | | | | | | | | |

**Figure 6: Solving the third part of the chocolate problem without dragging and with parameters: equation view**

To solve the first part of the seats problem, Salim used one row and, at the same time, parameters. Figures 7 and 8 describe his method.

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | price of front seat | price of rear seat | | overall known price | | overall computed price |
| 2 | 8 | 6 | | 650 | | 620 |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | number of front seats | number of rear seats | | total numberof seats | | |
| 7 | 10 | 90 | | 100 | | |
| 8 | | | | | | |

**Figure 7: Solving the first part of the seats problem in one row and with parameters**

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | price of front seat | price of rear seat | | overall known price | | overall computed price |
| 2 | 8 | 6 | | 650 | | =A7*$A$2+B7*$B$2 |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | number of front seats | number of rear seats | | total numberof seats | | |
| 7 | 10 | =D7-A7 | | 100 | | |
| 8 | | | | | | |

**Figure 8: Solving the first part of the seats problem in one row and with parameters: equations view**

To solve the second part of the seats problem, Salim also used one row and parameters. This time he changed the role of the previous unknowns (numbers of chairs) to become parameters, while the previous parameters (prices of chairs) turned to be unknowns. Figure 9 and Figure 10 describe Salim's solution.

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | price of front seat | price of rear seat | | overall known price | | overall comput ed price | |
| 2 | 12 | 9 | | 975 | | 975 | |
| 3 | | | | | | | |
| 4 | | | | | | | |
| 5 | | | | | | | |
| 6 | number of front seats | number of rear seats | | total number of seats | | | |
| 7 | 25 | 75 | | 100 | | | |
| 8 | | | | | | | |

**Figure 9: Solving the first part of the seats problem – changing the role of the unknowns and the parameters**

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | price of front seat | price of rear seat | | overall known price | | overall computed price |
| 2 | 12 | =(3/4)*A2 | | 975 | | =$A$7*A2+$B$7*B2 |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | number of front seats | number of rear seats | | total numberof seats | | |
| 7 | 25 | =D7-A7 | | 100 | | |
| 8 | | | | | | |

**Figure 10: Solving the first part of the seats problem – changing the role of the unknowns and the parameters: equation view**

**6.2 Categories of the examples given by the preservice teachers on parameters**:

Using content analysis, It was found that the examples on parameters given by the preservice teachers could be categorized in one of three categories:

a) A letter representing a parameter:
   Examples: each of the following answers was coded as a letter representing a parameter:
   - a and b in y=ax+b
   - a and b in ax=b
   - a and b in ax+b
   - The slope of the straight line

b) A number:
   Examples: each of the following items was coded as a number:
   - 3 and 5 in y=3x+5
   - 3 and 5 in 3x=5
   - 3 and 5 in 3x+5
   - 3

c) A letter representing a variable:
   Examples: each of the following answers was coded as a letter representing a variable:
   - a, b, c
   - x+y
   - x,y
   - $x^2+2x-5=0$
   - y=x

**6.3 Categories of the definitions given by the preservice teachers for the parameter**:

Using content analysis, I found that the definitions for the parameter given by the preservice teachers could be categorized in one of three categories:

a) A wrong definition: this is a definition which did not distinguish between the parameter and the variable concepts or the parameter and the unknown concepts.
   Examples: each of the following answers was coded as a wrong definition:
   - The parameter is a variable where we can substitute any number that we want.
   - The parameter is the unknown in an equation.

b) A right-wrong definition: this is a definition which included elements which could be associated with the parameter concept and elements which could be associated with the variable or the unknown concept.
   Examples: each of the following items was coded as a right-wrong definition:

- The parameter is a constant number multiplied by a variable.
- The parameter is a constant number that can have other values.

c) A nearly right definition: this is a definition with right elements but at least one element was missing.
Example: the following answer was coded as a nearly right definition:
- The parameter is this which we substitute in it to obtain a coefficient.

**6.4 Percents of the types of examples given by the preservice teachers on parameters**:
Table 1 describes the percent of examples given by the preservice teachers on parameters, before working in the spreadsheets environment, in each of the three categories described above. The table refers to each academic year in a different column.

**Table 1: Percent of examples on the parameter concept before working in the spreadsheets environment**

| The example type | Percent of examples in the academic year 2006-2007 N=3x27=81 | Percent of examples in the academic year 2007-2008 N=3x31=93 |
|---|---|---|
| A letter representing a parameter | 22.22% | 24.73% |
| A number | 27.16% | 22.58% |
| A letter representing a variable | 50.62% | 52.69% |

Table 1 shows that in both academic years less than forth of examples given by the preservice teachers, before working in the spreadsheets environment, were actually examples on parameters.

Table 2 describes the percentages of examples given by the preservice teachers on parameters, after working in the spreadsheets environment, in each of the three categories described above. The table refers to each academic year in a different column.

**Table 2: Percent of examples on the parameter concept after working in the spreadsheets environment**

| The example type | Percent of examples in the academic year 2006-2007 N=3x27=81 | Percent of examples in the academic year 2007-2008 N=3x31=93 |
|---|---|---|
| A letter representing a parameter | 62.96% | 67.74% |
| A number | 7.41% | 5.38% |
| A letter representing a variable | 29.63% | 26.88% |

Table 2 shows that in both academic years more than sixty percent of examples given by the preservice teachers, after working in the spreadsheets environment, were actually examples on parameters.

**6.5 Percents of the types of definitions given by the preservice teachers for the parameter concept**:

Table 3 describes the percents of definitions given by the preservice teachers for the parameter concept, before working in the spreadsheets environment, in each of the three categories described above. The table refers to each academic year in a different column.

**Table 3: Percent of types of definitions for the parameter concept before working in the spreadsheets environment**

| The definition type | Percent of definitions in the academic year 2006-2007 N=27 | Percent of definitions in the academic year 2007-2008 N=31 |
|---|---|---|
| A wrong definition | 59.26% | 54.84% |
| A right-wrong definition | 11.11% | 12.90% |
| A nearly right definition | 29.63% | 32.26% |

Table 3 shows that in both academic years nearly thirty percent of the definitions given by the preservice teachers, before working in the spreadsheets environment, were nearly right definitions.

Table 4 describes the percents of definitions given by the preservice teachers for the parameter concept, after working in the spreadsheets environment, in each of the three categories described above. The table refers to each academic year in a different column.

**Table 4: Percent of types of definitions for the parameter concept after working in the spreadsheets environment**

| The definition type | Percent of definitions in the academic year 2006-2007 N=27 | Percent of definitions in the academic year 2007-2008 N=31 |
|---|---|---|
| A wrong definition | 14.81% | 12.90% |
| A right-wrong definition | 29.63% | 22.58% |
| A nearly right definition | 55.56% | 64.52% |

Table 3 shows that in both academic years more than half of the definitions given by the preservice teachers, after working in the spreadsheets environment, were nearly right definitions.

**6.6 Development of the parameter concept as a result of working in the spreadsheets environment**:

At the beginning, Salim gave the following examples on parameters: a, 0, 2. These are wrong examples. After solving word problems with the spreadsheets, Salim gave the following examples on parameters: a in ax, where x is unknown and ax is a multiple of it; a and b in x+ax+(ax+b), where x, ax, ax+b are three related amounts, a and b in ax+b where ax+b represents a line.

## 7. Discussion:

This research wanted to indentify the methods used by preservice teachers who study mathematics and computers to solve mathematical word problems in the spreadsheets environment, how they utilize the spreadsheets potential of parameters if the situation requires this use, what examples they give on parameters before and after working with the spreadsheets, and whether their work in the spreadsheets environment develops their conceptions of parameters. A discussion follows regarding the research findings related to these issues.

### 7.1 Utilizing the spreadsheets potential to solve word problems:

At the beginning the preservice teachers utilized just one potential of the spreadsheets environment which was building by dragging columns that fulfill specific mathematical relations. Gradually they turned to few rows which they built without dragging, and then to one row. The few rows indicate that the preservice teachers reflected on their solution with every new row and tried to improve that solution. The one row made the spreadsheets environment an algebra environment which includes an equation which describes the mathematical relation in the word problem. The dragging method is similar to the one reported by Sutherland and Rojano [20] which was used by the primary school students who participated in their research to solve the field measurement problem. The few rows method is similar to the one reported by Ainley [3] to be used by primary school students to solve the fencing problem (see appendix II). These similarities between this research's findings and other researches findings point that the learning context influences the students' solutions. The last method which the preservice teachers used (the one row method which integrated the use of parameters) is unique for the preservice teachers and points at their expertise in the spreadsheets environment.

### 7.2 Utilizing the parameter option in the spreadsheets:

At the beginning the preservice teachers were not aware to the parameter potential in the spreadsheets though they learned in a previous course about the option of fixing a reference when copying and pasting. Confronting the issue of solving all the parts of a word problem in one spreadsheet configuration, and discussing how to build such a configuration in the classroom, made the preservice teachers search for spreadsheets potentialities which facilitate the building of such configuration. When they were introduced to the $ sign as the sign which turns a spreadsheets cell into a parameter, they valued this potentiality and started to use it. As a consequence of becoming aware of the parameter potentiality, the preservice teachers, as we saw Salim do, used this potentiality, also when the solution of the word problem did not necessitate the use of parameters.

### 7.3 Examples and definitions that the preservice teachers gave for the parameter:

The findings about the preservice teachers' conceptions of the parameter concept, before working with the spreadsheets to solve word problems, show that most of the preservice teachers who participated in the research did not differentiate between unknowns and parameters or numbers and parameters. This situation changed after working with the spreadsheets to solve word problems whose solution needed using the spreadsheets functions and potentialities. Before working in the spreadsheets environment, less than twenty five percent of the preservice teachers gave right examples on parameters, while after working in the spreadsheet environment; more than sixty percent of the preservice teachers gave right example on parameters. The same thing happened regarding the definitions: before working with the spreadsheets environment, almost fifty five percent of the preservice teachers in one year, and almost sixty percent of the preservice teachers in

another year, gave wrong definitions of the parameter, while after working in the spreadsheets environment, less that fifteen percent gave a wrong definition of the parameter.
The fact that the preservice teachers who participated in the research could not define the parameter before or after working with the spreadsheets environment in a fully correct way could be related to the complexity of the parameter concept and the difficulty to differentiate between the parameter concept and the variable, the unknown or the number concepts. Ursini and Trigueros [23], for example, found that, "in general, students had great difficulties in working with parameters but, when they could assign a clear meaning to them, their difficulties decreased" (p. 361). Here, working with spreadsheets, the students difficulty with the parameter concept decreased but did not disappear.

## 8. Conclusions:

The parameter concept has an important role in Algebra in particular and mathematics in general. Mathematics books do not treat this concept directly, and consider it as needed only for the sake of other mathematical objects, especially the equation or the function. This situation results in the blurriness of the parameter concept and thus it results in the difficulty of students to differentiate it from other mathematical concepts which can be represented by letters. Students can be aware of the difference between the different Algebraic concepts: the parameter, the unknown and the variable when they work in an environment where the borders between the different concepts are clear. The spreadsheets could provide such environment when the work of the students is accompanied by a discussion in the classroom, for example discussing with the students what it means that they look for a row which includes a number given in the word problem, and then for a cell value in the row which represents a mathematical object which the word problem requires us to find its value. This discussion would probably lead the students to the concept of the unknown. Dettori et al [7] say that the spreadsheets facilitate knowing the concept of variable and not the unknown. This claim is true if the teacher does not engage the students with a discussion regarding their different moves in the spreadsheets environment. This claim could be verified in a future research, specifically how the spreadsheets could be used to differentiate between the three mathematical objects: the variable, the unknown and the parameter.
Treating the parameter concept more directly could be done with the help of the spreadsheets. This research described how the spreadsheets environment helped the preservice teachers absorb the parameter concept and helped them start to differentiate between it and concepts of other mathematical objects. This use of the spreadsheets should be done when it benefits the learning of mathematics, as in the case that this research describes.

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## Appendix 1: Word problems used in this research

**'The measurement of a field' problem taken from Sutherland and Rojano (1993)**:

The exact wording of the word problem, as given to the preservice teachers, follows: We want to use the spreadsheets to solve the following word problem:

a) The perimeter of a rectangular field measures 102 meters. The length of the field is twice as much as the width of the field. How much do the length and the width of the field measure?
b) What happens if the perimeter of the rectangular field measures 162 meters and the relation between the length and the width of the field stays the same?
c) What happens if the perimeter of the rectangular field measures 128 meters but the length of the field is three times its width?
d) Find a method where you solve the three parts of the problem together?

**'The chocolates problem' taken from Sutherland and Rojano (1993)**:

The exact wording of the word problem, as given to the preservice teachers, follows: We want to use the spreadsheets to solve the following word problem:

a) 100 chocolates were distributed between three groups of children. The second group received 4 times the chocolates given to the first group. The third group received 10 chocolates more than the second group. How many chocolates did the first, the second and the third group receive?
b) What happens if the number of the distributed chocolates is 280? What happens if the number of the distributed chocolates is 460?
c) What happens if the number of the distributed chocolates is 650, the second group received as the first group, and the third group receives 50 chocolates more than the second group?
e) Find a method where you solve the three parts of the problem together?

**'The theatre problem' taken from Dettori et al. (1995)**:

The exact wording of the word problem, as given to the preservice teachers, follows: We want to use the spreadsheets to solve the following word problem:

a) In the city theatre there are 100 seats which are distributed in two areas: the front and the rear. The price of a front seat is \$8 and that of a rear seat is \$6. The overall income, when selling all the seats, is \$650. How many front seats and how many rear seats are in the theatre?

b) In a special happening in the city theatre which we described above, the tickets are more expensive than in regular happenings. the administration of the theater intends to increase the price of the chairs, on condition that the ratio between the front seat price and the rear seat price stays 4 to 3, and the overall income becomes \$975. What would be the price of a front seat and a rear seat?

c) What possible relations are there between the two parts of the question?

## Appendix II:

'The fencing problem' taken from Ainley (1996):

The exact wording of the word problem, as given to the preservice teachers, follows: We want to use the spreadsheets to solve the following word problem:

a) A farmer has 30 meters of flexible fence. She wants to make with it a sheep pen set against a wall, with the largest possible area. What would be the length and the width of the sheep pen?

b) What if the farmer has a fence of another length? Try at least three different lengths.