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Pre-service teachers' mathematical models' features

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Abstract
This study came to characterize the features of mathematical models built by mathematics pre-service teachers for a model eliciting activity. Fourteen groups participated in building the models. We used a combination of deductive and inductive content analysis to characterize the pre-service teachers' elicited models, taking into account features mentioned in the literature: content factors, representation form, operations, generalization, reality, accuracy, precision, robustness and fruitfulness. Moreover, two features were introduced by us, namely the interactivity and the finalization features.

The findings of the research indicate that models utilized mainly the list and text representations. Moreover, most of the participants' models had substantial accuracy and substantial preciseness. At the same time, most of the models had partial or no generalization, partial or no reality, partial or no interactivity and partial or no finalization. Furthermore, most of the models were without robustness and without fruitfulness.

Keywords: pre-service teachers, models' features, mathematical modelling

Introduction

Researchers point at the educational value of engaging students with modelling activities. For example, it helps students appreciate mathematics as a critical tool for analyzing situations and phenomena in their real world experience (Watters, English & Mahoney, 2004). Moreover, they point at the other learning aspects that are being taken care of when engaging students with modelling: the cognitive, the affective, the social, the behavioral and the meta-cognitive aspects (Awawdeh-Shahbari, Daher, & Rasslan, 2014; Daher, 2015). Specifically, researchers point at mathematical modeling as helping students develop various mathematical competencies, communication processes and social skills (Blum & Ferri, 2009; Lesh & Doerr, 2003).

Researchers in mathematical modelling have studied thoroughly the modelling processes or/and cycles in which students engage in order to build mathematical models suitable for the 'real life'/authentic problem/situation/phenomena (see for example Blum & Leiss, 2005; Daher & Awawdeh-Shahbari, 2015). The present study comes to examine a different perspective of the modelling activity, namely the features of the mathematical models elicited in order to represent situations or problems. Fourteen groups of pre-service teachers worked on a model eliciting activity named the 'summer reading' activity (English & Fox, 2005) to build a model suitable for assigning points to each student participating in the summer reading activity, for all of the books that the student read and wrote a report about the activity. The current research examines the features of the different models built by the pre-service teachers.

Theoretical background

The modelling approach:

The modelling approach emphasizes the mathematisation of realistic situations in a meaningful way to the learner (English & Fox, 2005). So, it can be said that the modelling approach emphasizes the effectiveness of the mathematics in the real life through suggesting modelling activities as reality-based contextual examples (Vorhölter, Kaiser & Ferri, 2014). In addition, these activities should include features that help to build models, such as quantities and operations (Doerr & English, 2003). Mousoulides (2009) gives examples on the types of quantities needed in modelling situations: accumulations, probabilities, frequencies, ranks, and gives examples on the needed operations: sorting, organizing, selecting, quantifying, weighting, and transforming large data sets.

Elicited models in modelling activities:

Models are conceptual systems (Lesh & Harel, 2003) that often represent generalizable ways of thinking (Lesh & English, 2005). Lesh and Doerr (2003) pointed at mathematical models as tools for describing, manipulating and controlling mathematical systems. Moreover, models can be in different forms and can be expressed using various representational media, including written and oral reports, computer-based representations, and paper-based diagrams or graphs (Lesh & Lehrer, 2003). Further, they may include symbols and spoken language (Lesh & Harel, 2003).

Elicited models' features:

Different ways are suggested in the literature to categorize and describe elicited models. Meyer (1984) suggested evaluating models according to the following features: accuracy (whether the output of the model is correct or very near to correct), descriptive realism (whether the model is based on correct assumptions), precision (whether the model's predictions are definite), robustness (whether the model is relatively immune to errors in the input data), generalization (whether it applies to many cases, and fruitfulness (whether the model's conclusions are useful or pointing to other situations).

On the other hand, Lesh and Lehrer (2003) suggested evaluating models according to the following features: the nature of their mathematical objects (quantities, shapes, locations), the nature of their mathematical relations among objects, the nature of their mathematical operations on objects, and finally, the nature of their mathematical patterns and regularities that govern the prior objects, relations, and operations. These structural elements enable the models to be mapped, applicable and to accommodate new situations (English, 2003).

A third way to categorize models is that of English and Fox (2005). They categorized the mathematical models built by middle school students for the summer reading activity according to the activity content factors, the representation form of the model and the mathematical operations that the model involves.

In the present study, we used all the features described in English and Fox (2005), as well as the features described in Meyer (1984) to characterize the models of the participating groups of students. The features taken from English and Fox (2005) are: (1) activity content factors, (2) the representation form of the model, and (3) the mathematical operations that the model involves. The features taken from Meyer (1984) are: (4) the accuracy of the model, (5) the generalization of the model, (6) the robustness of the model, (7) the descriptive realism of the model, (8) the preciseness of the model and (9) the fruitfulness of the model. In addition, we used two other features that we expected to shed more light on the models: interactivity of the model (how the model takes into consideration the interaction between the different activity content factors) and the finalization of the model (whether the model takes care of all the steps, including the last step).

The research goals and rationale:

Researchers in mathematical modelling have studied mainly the modelling processes and cycles in which students engage with when performing mathematical modelling activities. Few researchers were concerned with the features of mathematical models built by the students as a result of their engagement with mathematical modelling activity (Lesh & Lehrer, 2003; Meyer, 1984). The current research comes to shed more light on this issue. The results of the research would help mathematics teachers know and thus expect the features of mathematical models built by their students in the various levels. This will be done by comparing the features of models built by pre-service teachers, as reported in the present research, with the features of models built by primary school students, as reported in English and Fox (2005). This comparing will help maintain a wider picture of students' models in the various learning levels.

Research question:

What features do mathematical models of the summer reading activity have, when the models are built by mathematics pre-service teachers?

Method

Research Setting and Participants:

The current research examines the features of mathematical models built by pre-service mathematics teachers, when engaged with model eliciting activities. The participants were 67 pre-service teachers from three classes, who were engaged in building mathematical models fitting the "Summer Reading" activity. All the pre-service teachers worked in groups of 4-6, so there were 14 groups who worked in modelling the summer reading activity.

No participants had any previous experience of working with modeling activities. All the groups of pre-service teachers were asked to solve the summer reading activity (English & Fox, 2005). This activity included a scenario about a city council library conducting an annual summer reading program with prizes to be won. The scenario is shown in Appendix (1).

The pre-service teachers carried out the model eliciting activity as a task in a mathematics didactics course taught by the first author. The pre-service teachers decided with whom to work as well as when to carry out the activity, but not during the class time. So, they worked on the activity in groups without the interference of the researchers.

Data collecting tools:

The data, which served the current research, is the models developed by the groups of students' for the summer reading activity. These models were submitted on paper.

Data analyzing tools:

We used a combination of deductive and inductive content analysis (Cho & Lee, 2014) to characterize the pre-service teachers' elicited models of the summer reading activity. This use was encouraged because of the unique characteristic of content analysis being flexible in using "inductive or deductive approaches or a combination of both approaches in data analysis" (ibid, p.4). Utilizing the deductive content analysis, we took into account the categorizations found in the literature for mathematical models, specifically the three categories taken from English and Fox (2005). These categories are: content factors (number of books read, variety of books considered, reading level of the books, length of the books, student's grade level, and quality of written reports), form of representation (tables, lists, formulas, texts, flowcharts), and type of operations (assigning values or, using interval quantities, using weighting by assigning weight for every content factor from the overall score of the student, aggregating quantities by writing a formula to compute the score of all the factors). Other categories were taken from Meyer (1984): accuracy (the correctness of the output), generalization (the ability of the model to express many cases), robustness (whether the model is relatively immune to errors in the input data), reality (In our research, whether the model is based on assumptions that satisfy real life or not), preciseness (whether the model predictions are definite mathematical entities), and fruitfulness (whether the model points the way to other situations).

Utilizing the inductive content analysis to analyze the resulting models, we found that there is a need for two other categories that were not mentioned in the literature: the finalization of the model and its interactivity. In our case, the finalization of the model refers to whether the model considers the overall grade for a student or just the grade given to the student regarding one book. In addition it refers to whether the model considers the winner from the four grades or not). Moreover, the interactivity feature examines whether the model takes into account a specific feature in the light of another feature. In other words, the interactivity of the model refers to the ability of the model to represent the intersection between the various content factors of the activity: number of books read, variety of books considered, reading level of the books, length of the books, student's grade level, and quality of written reports.

In the present study, each one of the last above features has three values for a model. The accuracy of the model has the values: no accuracy (when there are at least two mistakes in the mathematical or spreadsheets operations performed), partial accuracy (when there is just one mistake in the mathematical or spreadsheets operations performed), and substantial accuracy (when there is no mistake in the mathematical or spreadsheets operations performed).The generalization of the model has the values: no generalization (when the model treats just one case), partial generalization (when the model generalizes one of the content factors), and substantial generalization (when the model generalizes more than one of the content factors). The robustness of the model has the values: not robust (when there is more than one input that is not taken care of by the model), partially robust (when there is only one input that is not taken care of by the model) and substantially robust (when all the input possibilities are taken care of by the model).The reality of the model has the values: not real (when there is more than one content factor not assigned values relying on assumptions related to real life, partially real (when there is only one content factor not assigned values relying on assumptions related to real life) and substantially real (when all the content factors are assigned values relying on assumptions related to real life). The precision of the model has the values: not precise (when there is more than one content factor whose predictions are not definite numbers but a set of numbers), partially precise (when there is only one content factor whose predictions are not definite numbers) and substantially precise (when all the content factors are assigned definite numbers). The fruitfulness of the model has the values: not fruitful (when more than one computation or equation do not lead to other situations), partially fruitful (when there is only one computation or equation that does not lead to other situations) and substantially fruitful (when all computations or equations point the way to other situations).The interactivity of the model has the values: no interactivity, partial interactivity (when there is interaction between just two of the content factors but not all of them), and substantial interactivity (when there is interaction at the same time between more than two of the content factors). The finalization of the model has the following three values: no finalization (the model describes just how to compute the score of one book), partial finalization (the model describes how to compute the score of the book and the reader), and substantial finalization (the model describes how to compute the score of the book and the reader and how to arrive at the competition winner).

Findings

The final models have been analyzed according to their features. We present here these features for the models.

All the content factors were observed in all the models. Regarding the representational forms, the models were built using five different representations: tables, lists, formulas, texts and flowcharts; some of the models integrated more than one form. Regarding the mathematical operations used in the models, the following were used: assigning values, using interval quantities, using weighting and aggregating quantities. The representational forms and the operations associated with the different produced models are presented in table 1.

Table 1: Representational forms and types of operations in the models

Group	Representation form					Type of operation			
	Table	List	Formula	Text	Flowchart	Assigning values	Using interval quantities	Using weighting	Aggregating quantities
1				√	√	√	√	√	√
2		√		√		√	√	√	
3			√	√		√		√	√
4		√	√			√		√	√
5	√	√				√	√	√	
6		√		√		√	√	√	
7	√					√	√	√	
8		√				√	√	√	
9		√		√		√	√	√	
10					√	√	√	√	
11		√				√	√	√	√
12		√	√	√		√	√	√	√
13	√	√	√			√	√	√	
14			√	√				√	√

Six of the other eight other features (Accuracy, generalization, robustness, reality, preciseness and fruitfulness of the model) of the models are described in Table 2, while the features (interactivity and finalization) are described in table 3.

Table 2: Models' features: Accuracy, generalization, robustness, reality, preciseness and fruitfulness

	Accuracy			Generalization			Robustness			Reality			Preciseness			Fruitfulness		
Model	without	Partially	Substantial	without	partially	Substantial	Without	partially	substantial	Without	partially	substantial	Without	partially	Substantial	Without	partially	Substantial
1			√		√		√				√				√			√
2			√	√			√				√				√	√		
3			√		√			√		√					√	√		
4			√	√				√				√			√	√		
5		√			√		√					√			√		√	

6		√			√	√		√				√		√	
7		√			√	√			√			√		√	
8		√	√			√		√				√	√		
9		√		√		√				√		√		√	
10		√	√			√			√			√			√
11	√			√		√		√			√		√		
12	√			√		√			√			√	√		
13	√				√	√		√			√		√		
14		√		√			√		√			√	√		

Table 3: Models' features: interactivity and finalization

	Interactivity			Finalization		
Model	Without	partially	substantial	Without	partially	Substantial
1	√			√		
2	√			√		
3		√			√	
4		√			√	
5		√		√		
6		√			√	
7			√	√		
8	√				√	
9	√				√	
10			√	√		
11	√			√		
12		√		√		
13		√			√	
14	√				√	

We give now two examples on students' models. Doing that, we also describe their features.

Model 1 (Model 5 in table1, 2 & 3):

The first model was built through one stage: writing the rules for computing the score of a student for each content factor. This says that the goal of the model was to put a basis for computing the scores associated with every content factor. Moreover, the model is composed of six tables: five for the factors, and one that takes care of fitting the score given for the book's grade with the student's grade. The following first table is an example on the tables defining the rules for computing the content factor's score, while the second table fits the score given for the book's grade with the student's grade.

The number of books read by a student

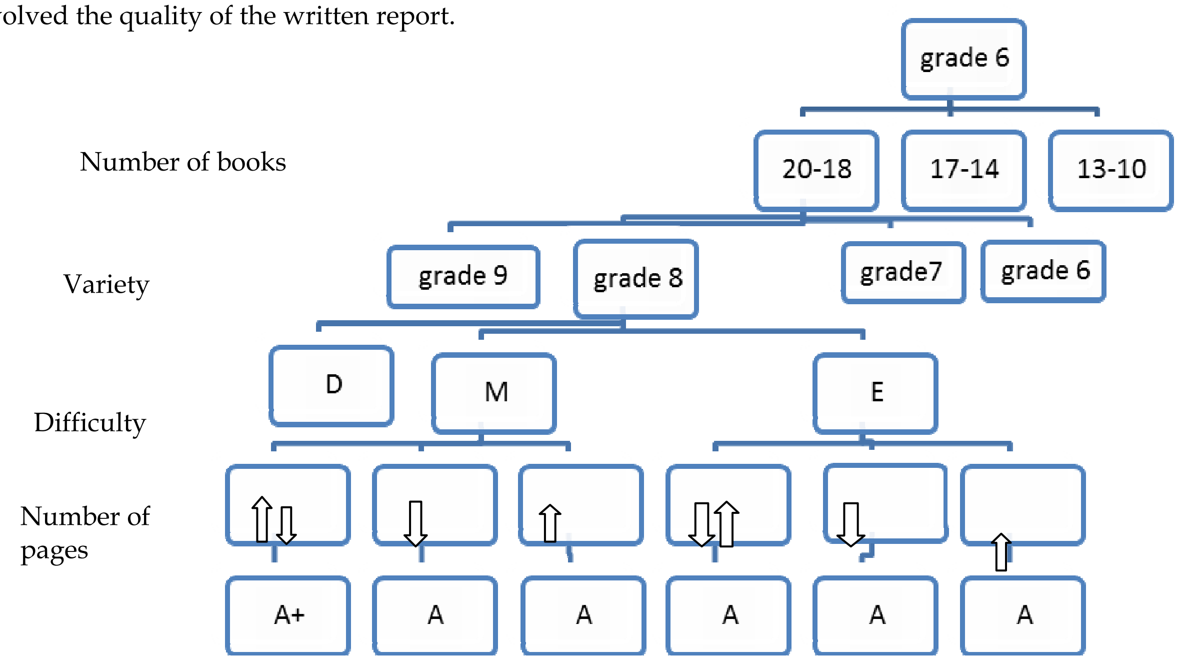
Number of books	10	11	12	13	14	15	16	17	18	19	20
Points	1	2	3	4	5	6	7	8	9	10	11

Fitting the score given for the book's grade with the student's grade:

The book's grade / The student's grade	6	7	8	9	10+
6	1	2	3	4	5
7	-1	1	2	3	4
8	-2	-1	1	2	3
9	-3	-2	-1	1	2

The first model could be characterized as:
A model that uses primarily the table representation, it takes into account all the content factors of the activity, and it is involved with the following mathematical operations: assigning values and using intervals. Moreover, this model can be categorized as substantially accurate, for there is one inaccurate mathematical operation performed in it in the table fitting the score given for the book's grade with the student's grade (in the second column of the book's grade, the difference between the student's grade should be 1 and not 2). The model can be categorized as substantially general, for it computes the score of every book and not just one book. It can also be categorized as partially robust for it gives the first ten books 1 point and the second ten books 10 points, which is not proportional. This model satisfies the reality feature substantially because it has no value not related to real life. The model also satisfies the preciseness feature substantially, for the scores computed for the different content factors have one value. Further, it is partially fruitful for giving a score for the variety of the books, only four types were considered. Furthermore, this model is not interactive for it does not compute the score of a content factor for the values of another content factor. There is interaction only between the student's grade and the book's grade, but this interaction has a correcting function only. Moreover, the model is not final at all, for it computes only the score of the book, but not for the student or to decide upon the winner.

Model 2 (Model 10 in table 1, 2 & 3):
The model consists of four flowcharts, where each grade had one flowchart consisting of five levels: the first level involves the number of the books, consisting of three domains: 10-13 books, 14-16 books and 17-20 books. The second level involved the variety, consisting of four values: grade 6, grade 7, grade 8 and grade 9. The third level involved the difficulty of the book, consisting of three values: D= difficult, M= middle and E=easy. The fourth level involved the number of pages in the book, consisting of three domains: less than 50 pages, 50-100 pages and more than 100 pages. The last level involved the quality of the written report.



Quality of the
written report

The second model could be characterized as:
A model that uses the flowchart representation, it considers all the content factors of the activity, and it is involved with the mathematical operations: assigning values, using weighting and using intervals.

The model has a partial accuracy for it did not take into consideration all the values of the quality of the written report. On the other hand, this model has substantial generalization for it is written in symbols and letters. At the same time, the model satisfies partially the robustness feature, because the model takes into consideration only two levels of the quality report's score (A+ and A). Moreover, the model has partial reality for it described books read by the students, where the book lengths are hundreds of pages. The model has also substantial preciseness, but partial fruitfulness, where the partial fruitfulness is a consequence of the extension of the model may need more than three levels for the number of page or difficulty as the present model suggests. Furthermore the model is not final because it computes only the score for each book, but not for each student or to decide upon the winner. At the same time, the model has a substantial interactivity because to compute each book's score, you do not compute the score for each factor separately, but it is necessary to move through a path in which you compute the score for a content factor for each value of the preceding content factor.

Discussion

The current research wanted to examine the features of students' models when working with a model eliciting activity. Discussing our findings, we first discuss the features taken from English and Fox (2005), and then those taken from Meyer (1984) and the two arrived at by us.

Content factors considered in the models:
All the models built for the 'summer reading activity' by the participants took into consideration the five content factors described in the activity text: the variety of the books, the number of the books, the number of a book's pages, the level of a book, and a book's summary quality. These results are not in line with the results of English and Fox (2005) who reported that seventh grade students' models of the summer reading activity did not take into consideration all the content factors in the activity, and only one model took into consideration all the content factors. The difference in the results is clearly due to the participants' mathematical and problem solving experience, where the participants in our case were college pre-service teachers who had good experience in mathematical problem solving.

Representational forms of the models:
The results indicate the prevalence of the lists and texts representation in the models. We should keep in mind, regarding the models' representation forms that the main function of the text was to describe the method of computations to arrive at the winner, while the main function of the other forms of representation was to arrange the data and allow its computation, in order to arrive at the winner. This is one reason that the models included the text as additional representation of the mathematical models that modeled the reading summer activity.

The use of lists to arrange the data of the summer reading activity indicates the functional role of this representation as graphic organizer that assisted the pre-service teachers to sort, classify, and connect information (Ontario Teachers' Federation (OTF), 2013). This supported the pre-service teachers in organizing the whole data in the activity and thus to arrive at decisions regarding the relations among the various content factors. In the findings of English and Fox (2005), all the groups of students used texts as representational form for their data and decisions, while the next prevalent representational form was formulae. In the present research, the formula was induced in the other representational forms, especially tables and lists.

Operations utilized in the models:
Some specific types of operations were used in constructing the models. These operations included: assigning value points, using interval quantities, using weighting and aggregating quantities. These operations used in the students' models are influenced by the content of the mathematical activity, so we find the same type of operations in English and Fox (2005). This influence of the activity features on students' outcome of learning has been reported in the literature, for example in Daher and Baya'a (2012).

Now we turn to the features taken from Meyer (1984) and the two suggested by us.

Accuracy and Preciseness:
Most of the models had substantial accuracy and substantial preciseness. The substantial accuracy and substantial preciseness could be related to the participants were mathematics pre-service teachers. The mathematics pre-service teachers carried out a modelling activity that required primary school arithmetic's operations, resulting in substantial accuracy. The substantial preciseness resulted from the pre-service teachers' specialty, which is mathematics education. This specialty supported the pre-service teachers in suggesting models with definite mathematical entities. It could be argued that the characteristics of the learners affected to a large extent their learning activity in general (Nakayama, Yamamoto, & Santiago, 2007) and in mathematical activity in particular (Motley-Locklear, 2012).

Generalization, reality, interactivity and finalization:
Most of the models had partial or no generalization, partial or no reality, partial or no interactivity and partial or no finalization. The little prevalence of substantial generalization could be due to the little experience of the pre-service teachers of carrying out modelling activities. This little experience made the participants not aware to the importance of generalization in activities that came to model authentic mathematical phenomena (English, 2003; Lesh & Harel, 2003). It is expected that more experience of the participants with mathematical modelling would improve their attention to generalization.

The little prevalence of substantial reality in the models could be a result of the participants taking into consideration the mathematical aspect of the modelling activity more than the authentic aspect of it. Here too, more experience with modelling activities would make learners more aware to the need of combining the mathematical and authentic aspects together in their suggestion of models appropriate to the mathematical authentic activities. This need for experience in modelling activities is mentioned by researchers (e.g., Lesh and Lehrer, 2003)

The little prevalence of substantial interactivity in the models could be due to the complexity of the activity, where various content factors were involved. This variety of the content factors made the participants try to take care of every content factor but not the interaction of them.

The absence of substantial finalization could be due to the participants taking care of computing the scores of the content factors but not the student or the winner. This neglect of the student or the winner points at the need of the pre-service teachers' discussions of their resulting models after they built these models. This discussion could present a platform for the learners to describe, argue and reflect about their models (Shemwell & Furtak, 2009), what supports the attention to missing components of the models.

Robustness and fruitfulness:
Most of the participants' models were without robustness and without fruitfulness. The little prevalence of robustness is due, as argued above, to the complexity of the activity, where this complexity constrained the participants' ability to take into consideration all the inputs related to the different content factors. This influence of the features of the activity on students' learning outcomes is pointed at by researchers (e.g., Lim & Morris, 2009).

The little prevalence of fruitfulness could be a result of the participants' inclination to simplify the mathematical situation, so they did not take into account all the possible values of the content factors related to the situation.

Conclusions

It was the goal of the present research to analyze the features of mathematical models developed by pre-service teachers for the summer reading activity. This analysis was done utilizing two frameworks: English and Fox (2005) and Meyer (1984). The research findings indicated the influence of the participants' characteristics, their learning experiences and the activity features on the participants' learning outcomes, in our case students' models.

The influence of the participants' characteristics and the activity features on the features of the participants' models points out that modelling activities should be chosen carefully, so they fit the participants and so that their complexity increases gradually.

The influence of the participants' experiences, in our case in modelling, on their learning outcomes, indicates that modelling activities should be introduced to students in a sequence of activities, rather than in a singular activity.

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Appendix (1)

The scenario of the 'Summer Reading Actitiy'as in English & Fox (2005):

Information: The Brisbane City Council Library and St. Peters School are sponsoring a summer reading program. Students in grades 6-9 will read books and prepare written reports about each book to collect points and win prizes. The winner in each class will be the student who has earned the most reading points. The overall winner will be the student who earns the most points. A collection of approved books has already been selected and put on reserve. See the previous page for a sample of this collection. Students who enroll in the program often read between ten and twenty books over the summer. The contest committee is trying to figure out a fair way to assign points to each student. Margaret Scott, the program director, said, "Whatever procedure is used, we want to take into account: (a) the number of books, (b) the variety of the books, (c) the difficulty of the books, (d) the lengths of the books, and (e) the quality of the written reports.

Note: The students are given grades of A+, A, A-, B+, B, B-, C+, C, C-, D, or F for the quality of their written reports.

Your mission: Write a letter to Margaret Scott explaining how to assign points to each student for all of the books that the student reads and writes about during the summer reading program.