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Assessment of uncertainty propagation using first-order Markov chain for maintenance of pavement degradation

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ABSTRACT

The first-order Markov Chain (MC) is used to predict the degradation of three types of pavements (rigid, semi-rigid, and mix) utilising database in the five departments in the West of France. The assessment of uncertainty in the MC evolution is presented through studying the trend of mean and standard deviation, for components of the transition probabilities (TP) using different time steps (2, 3, 4, 5 and 6 years). The results show that the trend of rigid pavements is constant with time in terms of coefficient of variation. For semi-rigid and mix pavements, the trend of the standard deviation was constant with time. These statistical properties offer the opportunity to provide uncertainty modelling of TP. The propagation of uncertainty for 2 and 6 years time steps through the prediction of pavement condition index is also performed for analysing the effect of the uncertainty. We compare the profile of states obtained from each time step in view to analyse the short (2 years) and medium term (6 years) potential of prediction.

1. Introduction

1.1. Background

For short-term planning and with limited budgets specified for maintenance works, it is advisable to use preventive maintenance strategy based on the condition state of the infrastructure. The use of such strategy usually contributes in extending the life cycle of pavement section (Bekheet *et al.* 2005), but not the structure of the road. It is well recognised that reliability and risk analysis offers the theoretical framework for maintenance optimisation. It requires probabilistic models of degradation.

In this paper we use the first-order Markov Chain (MC) probabilistic model to predict the degradation of three pavement types; rigid, semi-rigid, and mix, in the West departments of France. The total number of sections is 12,256 distributed among the three different types of pavements. The assessment of uncertainty in the MC evolution is presented through studying the trend of mean and standard deviation. These statistical parameters offer the opportunity to provide uncertainty modelling of transition probabilities (TP) for different time step. The first-order MC property is also checked through performing the state transition sequences (STSs) and the chi-square test. We want to assess good Markov model for the different steps of degradation of the pavement asset through concentrating on preventive maintenance, which is presented by condition state 2 or 3 (good or fair). As the preventive maintenance is applied in good or fair condition of the asset, then the indirect cost of the road user represented by the delay (additional travel time) will be less than in case of long-time corrective maintenance which will increase the delay of

the road user and consequently increased the indirect cost of maintenance works.

The main objectives of the paper are:

(1) to provide the MC for pavement maintenance through the identification and calculation of the transition probability matrix (TPM). The TPM is a square matrix that is composed from $n \times n$ cells, where n is the number of condition states. In each row of the matrix, two probabilities are considered except for the first row all probabilities are calculated; P_{11}, P_{12}, P_{13}, P_{14}, and P_{15}. For the remaining rows, the probability to stay in the same condition (diagonal terms; P_{22} for example to stay in the good condition) and the probability to move to the next condition (P_{23} for example from good to fair condition) are calculated.
(2) to analyse the effect of the time step for TPM computation, five time steps (transitions) are computed: 2, 3, 4, 5 and 6 years.
(3) to apply the analyses on the three different pavement types and model the uncertainty of assessment depending on the time-step.
(4) to analyse the effect on condition state distribution with time.

Our paper is composed from seven sections. The first section is an introduction. The second section shows the data base description and main assumptions used for MC computations. The third section illustrates checking MC property. Effect of time step and computation of different time steps TPMs is presented in Section 4. Section 5 shows a comparison between the three pavement types in terms of uncertainty. Section 6

illustrates the uncertainty propagation comparison between 2 and 6 years time steps. Finally, Section 7 presents the effect on condition state distribution with time.

1.2. Beyond state of the art

Pavement performance is essentially concerned with the prediction of the future pavement condition for the purpose of maintaining and management the pavement network (Abaza 2015). Pavement performance has been traditionally presented using a performance curve which shows how the pavement condition declines over time in the absence of maintenance and rehabilitation works (Shahin 1994, Huang 2004). The pavement condition has been historically presented by indicators such as pavement condition index (PCI), present serviceability index, and distress rating (AASHTO 1993, Shekharan and Gilman 2005, ASTM 2018, Shah *et al.* 2013). The emphasis on developing effective models for predicting pavement performance had become of great importance with the merge of pavement management science over three decades ago. Therefore, an effective pavement prediction model is a significant component of any advanced pavement management system (PMS) (Abaza *et al.* 2004, Jorge and Ferreira 2012, Khan *et al.* 2014).

Generally, there are two types of models used in predicting future pavement conditions: deterministic and stochastic models (Wang *et al.* 1994, Li *et al.* 1997, Amin 2015). Both types of model can predict the future pavement conditions; however, the stochastic-based models have gained wider use in pavement management applications because it makes possible the risk assessment (Hong and Wang 2003). This can be attributed to the fact that pavement performance has been identified as random which requires assigning different levels of uncertainty (i.e. probability) to different pavement condition outcomes (Abaza 2015). The stochastic model that was used by several researchers over the last three decades had mainly relied on deploying different forms of the discrete-time Markov model (Butt *et al.* 1987, Li *et al.* 1996, Hong and Wang 2003, Abaza 2006, Abaza and Murad 2009, Jiang 2010, Mandiartha *et al.* 2012, Lethanh and Adey 2013). The homogenous (steady-state) and non-homogenous (deploys different TP for each transition) chains are the two most popular forms of the deployed discrete-time Markov model (Abaza 2015). In our paper, the homogeneous MC is selected and shown to be a good candidate for the modelling of the degradations processes we are facing. That is based mainly on the assumption that the traffic is almost of similar type and frequency with time.

Homogenous MC is one of several methods that is devoted to the prediction of the future pavement condition as well as used in several PMSs (Rayya *et al.* 2014). The use of MC requires calculating the TPM from the available historical data of pavement. The MC theory relies on a discrete probabilistic approach that is widely used in developing probabilistic models for predicting pavement deterioration (Rayya *et al.* 2014). According to Rayya *et al.* (2014), many researchers have selected MC in their work (Butt *et al.* 1987, RIMES 1999, Silva *et al.* 2000, Kamalesh 2009, Costello *et al.* 2011, Uchwat and MacLeod 2012). In all these studies, there is an agreement that the principles and conditions of the MC theory

(stochastic process is discrete in time and a finite state space) are applicable to pavement deterioration modelling. This is based on the fact that it is common to analyse road network condition at specific points in time (e.g. annually) and with visual inspection, and the number of conditions states can be made finite by defining a limited number of condition states for the defect being modelled. Additionally, the first-order Markov property (future state of the process depends on its current state and not past state) holds usually in pavement deterioration. That means that the present condition contains implicitly a given history (load and resistance) or that whatever the history, a given present distribution turns always to a given future distribution.

Through the literature, MC model is also used to predict the future condition and performance of pavements by many researchers (Hong and Wang 2003, Peirce 2003, Aloysius and Diah 2005, Abaza 2006, Osama and Lana 2007, Abaza and Murad 2009, Mandiartha *et al.* 2012, Uchwat and MacLeod 2012, Katkar *et al.* 2013, Lethanh and Adey 2013, Rayya *et al.* 2014). All these studies show using first-order MC from two consecutive inspection years without considering uncertainty of estimation, but our paper deals with calculating the TP from age of pavement sections, which leads to provide an uncertainty modelling of the TP. The form of the TPM used in the paper is composed from the diagonal probabilities in addition to extra probabilities concerning the first row of the TPM as follows:

$$
\text{TPM} = \begin{bmatrix}
p_{11} & p_{12} & p_{13} & p_{14} & p_{15} \\
0 & p_{22} & p_{23} & 0 & 0 \\
0 & 0 & p_{33} & p_{34} & 0 \\
0 & 0 & 0 & p_{44} & p_{45} \\
0 & 0 & 0 & 0 & p_{55}
\end{bmatrix}.
$$

The most highlighted and interested points presented in the paper are: using the homogenous first-order MC as a stochastic model in prediction the performance of three different pavement types (rigid, semi-rigid, and mix) by calculation the TPM for different time steps (2, 3, 4, 5 and 6 years); uncertainty assessment using statistical parameters and comparison for t three pavement types.

2. Database description and assumptions use for MC computations

2.1. The data base components: when, where and what?

The data collected for the Quality Index of National Roads (IQRN) campaign are described in the Data Collection Method LPC n° 38–2 (LCPC 1998). The database represents the pavement conditions for about 4712 sections distributed among three types of pavements; rigid (2129 sections), semi-rigid (1630 sections) and mix (953 sections), each section of 200 m length distributed in the five north-west coastal departments of France (22, 29, 35, 44, and 56). The database is obtained from IFSTTAR (the French institute of science and technology for transport, development and network). The main indicators in the database were the Patrimony Note (NP) which is equivalent to the PCI, the structure type (rigid, semi-rigid, and mix), survey year (inspection year) and the year of wearing course in

the IFSTTAR Database build in 2015. Types of pavements are defined hereafter:

Rigid pavement/concrete pavement:

This pavement is substantially constructed of cement concrete. The underlying courses may be treated with special cementitious road binder or without any binder (aggregate alone).

Semi-rigid pavement:

This pavement is a composite pavement material composed of a bituminous surfacing and one or more courses that are treated with cementitious binders and which make a significant structural contribution. The porous asphalt has air voids between 25% and 30% (by Marshall mix design volume) filled or flooded by special formulated high performance polymer modified cement mortar material. These courses can also be treated with hydrocarbon binders and which by their stiffness or thickness cannot be considered as structurally flexible, e.g. a bituminous concrete layer of 8 cm (rolling layer) on a bituminous base course of 16 cm (aggregate treated with a bituminous binder) on possible other layers with or without bituminous binder.

Mix pavement:

This pavement has an upper part made of bituminous layers (often only one, the rolling course) relying on a set of cement treated layers, often only one layer, which is thick enough to give stiffness to the pavement.

For each section and according to the type of pavement, the NP rating is evaluated and accordingly five condition states are identified (Very Good (VG), Good (G), Fair (F), Poor (P) and Very Poor (VP)). The number of sections in each condition state for each type of pavement was:

- for rigid pavements (2129 segments): 1494 VG, 355 G, 207 F, 73 P.
- for semi-rigid pavements (1630 segments): 1317 VG, 204 G, 89 F, 20 P.
- for mix pavements (953 segments): 782 VG, 116 G, 46 F, 9 P.

The age of each section is calculated based on the difference between the date of surveying, here 2003, (inspection) and the date of wearing course up to 15 years old. Table 1 illustrates the condition states and corresponding NP values. The integer value and the scaling comes from French institute of science and technology for transport, development and network (IFSTTAR) because when it set up the rules (and it is still the case now), it was preferred to provide to inspectors a wide range of integer numbers than the same range (for instance 0–2) with decimal numbers. The reason comes mainly from the risk of errors in the reporting. That is shown to be the main human factors, when building a data base from outdoor visual inspections Another point is the more the scale is detailed (for

instance from 3 [0 10 20] to 5 [0 5 10 15 20]) the more time it requires. That is why this scaling is a kind of compromise between the risk of reporting, the time and the accuracy of the measurement from visual inspection. It is conserved until now for reasons of consistency even in some cases, numerical image post-treatment provides a better accuracy (O'Byrne *et al.* 2013, 2014). Anyway, the results obtained from these recent technical developments can be rated within a less accurate scaling.

2.2. Assumptions

The following are the main assumptions that are used in the analysis of the database for the different pavement types in order to calculate the MC TPM for different time steps:

- Almost the same traffic – the roads having the same role in the French network, this assumption is realistic;
- Given the type, the building family (structural design, thickness of layers, quality of works, applied standards) is similar; This assumption is realistic because in the time period of building or renewal (15 years) before the survey, there was no significant change in the design standard or material improvement for the roads belonging to this secondary network;
- Same climatic conditions; That is the case because departments are located in the North West part for France and they are all costal: that means the temperature in winter and summer, the number of freezing days and the rainfall are very close to each other.

These assumptions allow us to rank the sections based on homogeneous degradation process. As a consequence, the state of section i at time $t + 1$ describes accurately the possible condition state of section j at the same time: we assess TPM even if a given section is not observed every year.

A stochastic process is a Markov process if it satisfies the following condition: given that the present (or most recent) state is known, the conditional probability of the next state is independent of states prior to the present (or more recent) states (Katkar and Nagrale 2014). The homogenous MC is a stochastic process with the following properties:

- Discrete state space,
- Markovian property, and
- One step TP that remains constant over time (homogeneity).

If additionally the discrete state space has finite number of states, then it is termed as finite state MC, we use the term MC for finite MC in the following. Homogenous MC is completely determined once the transition matrix and sets of unconditional probabilities for initial states are specified. Knowledge of these two sets of probabilities allows the probabilistic prediction of specific states at future times.

2.3. Verification of Markovian property with STSs

If the Markov property holds then $P(j, m \mid i) = p_{jm}$, namely, the probability of going from state j to state m, given state i

Table 1. Pavement condition states and corresponding NP values (Lepert *et al.* 2000).

No.	Condition states	NP (PCI)-rating
1	Very good	20
2	Good	19
3	Fair	17–18
4	Poor	14–16
5	Very poor	<=13

occurred previously to j $(i \geq j \geq m)$. An analysis using available data is developed in the following sections to illustrate the verification of the Markovian property. The MC is applied to the available database based on pavement section age up to 15 years. Each of the cases described hereafter involves analysis of two three-STSs. A three-STS consists of three condition states: past, present and future. These three states correspond to three consecutive pavement conditions rating occurring over a 2 years period to ensure a minimum time interval for detecting a change in condition from visual inspection and reduce the effect of error assessment for small changes. Two possible transition sequences, with the same present and future states but different past states are tracked to illustrate if there is a difference in occurrence dependent of past state history. A frequency analysis of sequence occurrence is employed. If there is no significant difference in frequency between the sequences being tracked for all the transition sequences, this may indicate the Markovian property satisfied. Transition sequences of the condition states most frequently occurring in the database are assumed sufficient to establish the Markovian property as it applies to the entire deterioration model. The following terminologies are required for the informal analysis of STSs based on frequency probabilities (Katkar and Prashant 2014):

- STS refers to a particular three states sequence of concern. This sequence incorporates three consecutive condition ratings of a pavement section to establish past-present-future identification.
- State sequence occurrences (SSO) refer to the number of times a specified STS appears in the available database.
- Two-state occurrences (TSO) refer to the number a specified two-state sequence appears in the available database. The two-states sequence involves the past state and the present state.

Tracking these occurrences allow for the generation of frequency probabilities as will be described later; for example two possible STSs are:

(1) $(4,4 \mid 5)$: past = 5, present = 4, future = 4. $[(G, G \mid VG)$: past = VG, present = G, future = G]
(2) $(4,4 \mid 4)$: past = 4, present = 4, future = 4. $[(G, G \mid G)$: past = G, present = G, future = G]

The two-states sequence of concern for $(4,4 \mid 5)$ is the transition from state 5 to state 4 while for the two-states sequence of concern for $(4,4 \mid 4)$ state 4 remains unchanged from the past to the present.

2.4. Computation of SSO and TSO from database

To illustrate the concept of frequency probability, an example using real database is presented in Table 2. The frequency probability ratio (SSO/TSO) of the two states transition sequence for the three pavement types is computed in Table 2. If the Markovian property holds, the probability of this transition should be independent of past states. From Table 2, taking rigid pavements as an example, the probability of transition from state 5 (past) to

Table 2. Frequency probabilities to test Markovian properties (based on our database).

Pavement type	State transition (1)	(SSO/TSO)	State transition (2)	(SSO/TSO)	$|\Delta|$
Rigid	$(4, 4 \mid 5)$	0.5877	$(4, 4 \mid 4)$	0.6059	0.018
Semi-rigid	$(4, 4 \mid 5)$	0.0902	$(4, 4 \mid 4)$	0.099	0.0089
Mix	$(4, 4 \mid 5)$	0.8432	$(4, 4 \mid 4)$	0.8497	0.006

state 4 (present) and future (4) is 0.5877. Thus there is a 59% chance of maintaining state 4 if the previous state was 5. Using 4 as the past state, we have $P(4,4 \mid 4) = 0.6059$. Now as both probabilities are quite close, with difference $|\Delta|$ of 0.018, the Markovian property is achieved. The same procedure is followed for the remaining two types of pavements and differences are 0.009 and 0.006 for semi-rigid and mix pavements, respectively. It is probably of the same of magnitude than the statistical error but it will be checked in the future.

From this table, it is clear that there is no significant difference (slight and acceptable difference) in the frequency probabilities between the two states transitions for the three pavement types. The results of this analysis illustrate the independence of state-to-state deterioration transition from past state history for the rigid pavement and are less relevant for the two others.

Once the transition matrices are generated, the application of MC can be employed to predict deterioration overtime. In our case, the time steps are 2, 3, 4, 5 and 6 years as the deterioration process for pavement is slow and not practical to take 1year time interval (see justification in section 2.3). The multiplication of the transition matrices allows for the probabilistic predictions of future pavement conditions. If no maintenance is performed, the pavement section is eventually deteriorated to condition level #1.

3. Effect of time step (computation of different time steps TPMs) for rigid pavements

3.1. Effect of time steps

Increasing time step is more accurate for slow degradation process and human evaluation of condition state. Moreover, the process is less costly in terms of assessment when the time step is increasing whereas its use in a maintenance optimisation system leads to less accurate results (larger time discretisation). The effect of increasing time step on the accuracy of prediction must be quantified to discuss its benefit in maintenance optimisation. That is the objective of this section. To illustrate the effect of time step, five time steps are used in calculation the TPM: 2, 3, 4, 5 and 6 years. The TP values for rigid pavements for the 2and 6 years time steps are illustrated respectively in Figures 1 and 2 by considering 13 and 9 starting points respectively knowing the Markov property. The tabulated data are available and upon request from authors.

3.2. Comments on the results

As a general note, it is clear from Figures 1 and 2 that the degree of variation is different between the five condition states. The trend of the very good condition is the more stable compared
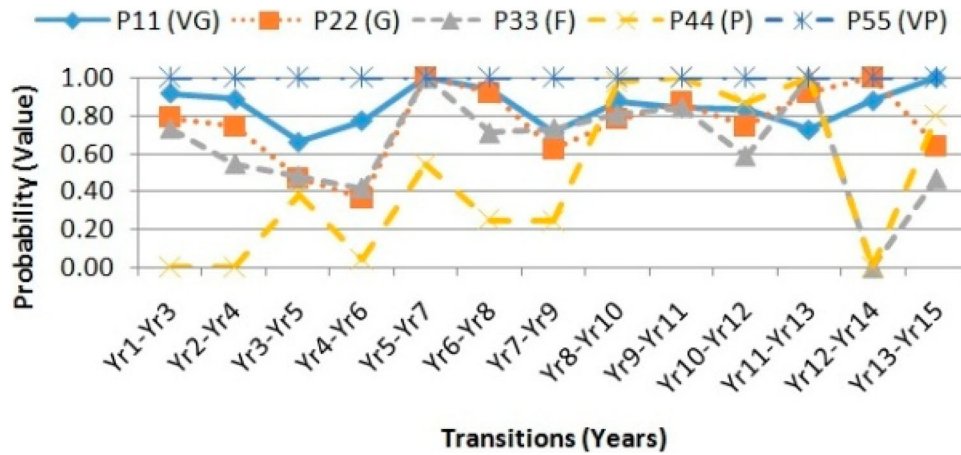
Figure 1. Deterioration of rigid pavements for all condition states based on 2 years transition cycle.
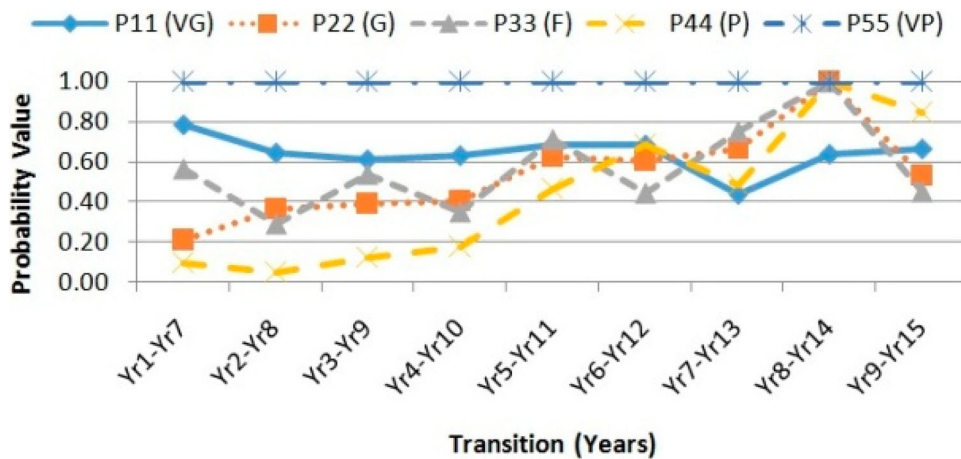

Figure 2. Deterioration of rigid pavements for all condition states based on 6 years transition cycle.

with other conditions: for example between 0.66 and 1 in Figure 1. On the contrary, the poor condition has the biggest variation among the five conditions with range between zero and one during the 15 years. The increasing of the scatter (except for the VP) comes first from the assessment of condition state and second from the database where all sections are assumed to be similar and to represent another section at the same age. The good and fair conditions vary from 0.2–1. The TP values for the different time steps for semi-rigid and mix pavements are available and upon request from authors. From Figure 2, we observe first that the probabilities to stay in the same stay decreases with the time step. Second, the trends are more regular leading to conclude that the scatter observed from smaller time steps is mainly due to the difficulty of assessment: it is easier to assess a change in condition state for larger time steps where degradations are more severe.

4. Comparison between the three pavement types in terms of uncertainity of assessment

To explore in detail, quantify and model the phenomenon identified in the previous section, we focus here on the computation of the uncertainty of assessment. In order to analyse the influence of time step on the assessment of TP, three statistical parameters; mean (μ), standard deviation (σ), and coefficient of variation (CoV) (σ/μ) are calculated based on the probability to stay in the same condition state for the five time steps (2, 3, 4 5 and 6 years). Probabilistic modelling from statistics requires some invariant with time or relationship between statistics and time. For degradation processes, probabilistic models relies usually on a constant CoV or a constant standard deviation with time. A constant CoV for a constant average means that the scatter will increase with time in comparison with a constant standard deviation. That leads to a more rapid decrease of the reliability, for a given mean value. That is why it is important to prove this property and we are looking for this invariant for each of the three types of pavements. For MC models, this property is investigated both in terms of error of evaluating the TPMs (section 4 and 5) and error of modelling in comparison with experimental data (section 6). It is only illustrated with the probability of very good condition (P_{11}) for three types of pavements (rigid, semi-rigid, and mix).

4.1. Rigid pavements

Table 3 presents the values of the three statistical parameters in the 15 years and Figure 3 illustrates the trend of the three parameters with the time steps graphically. From the table and

figure, it is clear that the mean value is decreasing with increasing time step, which meets the physical meaning that the probability of the very good condition state of pavement section is decreasing with the increasing of time step. From the evolution of standard deviation, the probability of assessment of pavement section is shown to be more accurate when increasing time steps (5 years for example), meaning that the uncertainty of assessment decreases when the time step increases. Finally the CoV trend is almost constant. This property is very important in terms of probabilistic modelling: we will consider the CoV as constant.

4.2 Semi-rigid pavements

Table 4 presents the values of the three statistical parameters depending of time steps and Figure 4 illustrates the trend of the p_{11} (avg.) and σ_{11} parameters. Note that the number of segments being in good condition at time 10 and staying in this state 2 years after is small for semi-rigid pavement and the uncertainty on the estimation of σ_{11} is significant. We plot on Figure 5 the evolution of p_{11} depending of the age for time-steps 2–6. Due to the increasing scatter after 6 years, we decided to compute p_{11} from the 6 first years ($n_p = 6$), leading to a significant uncertainty on σ_{11}. We add in Table 4 and Figure 4 the upper and lower bounds of the 95% Confidence Interval (CI) of σ_{11} computed from Equations (1) and (2); Alpha (α) is 0.05 for 95% confidence. The CoV is not reported because its evolution is not constant and consequently it doesn't help in modelling. The lower and upper intervals are calculated as presented in Equations (1) and (2).

From Table 4 and Figure 4, it is clear that the mean value is decreasing with increasing time step, which meets the physical meaning. For the standard deviation, the trend stays in the

Table 4. Mean, standard deviation and corresponding CI of p_{11} for 5 time steps (semi-rigid pavements).

Time step (Δt)	p_{11}(avg)	σ_{11}	n_p	Lower bound of CI of σ_{11}	Upper bound of CI of σ_{11}	CI
(2-years)	0.9011	0.0718	6	0.0448	0.1761	[0.0448–0.1761]
(3-years)	0.9006	0.0950	6	0.0593	0.2330	[0.0593–0.2330]
(4-years)	0.8645	0.0856	6	0.0534	0.2100	[0.0534–0.2100]
(5-years)	0.8462	0.0906	6	0.0565	0.2222	[0.0565–0.2222]
(6-years)	0.8074	0.0441	6	0.0275	0.1081	[0.0275–0.1081]

confidence interval of statistical uncertainty and no evolution can be shown: a linear fitting gives the equation $\sigma_{11} = -0{,}006 \Delta t + 0.0954$. We model the trend of the standard deviation as constant with value: 0.0718. There is no reason why the property should be the same for all the types of pavements. Scatter of the degradation process of semi-rigid pavements is shown to be stable with time when it was decreasing for rigid ones.

The following equations are used to calculate the lower and upper confidence intervals.

$$\text{Lower bound} = \sigma_{11} \times \text{SQRT}\left[\frac{(n_p - 1)}{\text{CHIINV}((\alpha/2), n_p - 1)}\right], \quad (1)$$

$$\text{Upper bound} = \sigma_{11} \times \text{SQRT}\left[\frac{(n_p - 1)}{\text{CHIINV}(1 - (\alpha/2), n_p - 1)}\right]. \quad (2)$$

4.3. Mix pavements

Table 5 presents the values of the three parameters in the 15 years and Figure 6 illustrates the trend of mean and standard deviation. For the same reason as previously (semi-rigid pavements), we report in Table 5 and on Figure 6 the lower and upper bounds of confidence interval for σ_{11}. Note that n_p is here decreasing with time step because it was more difficult

Table 3. Mean, standard deviation and CoV of p_{11} for 5 time steps (rigid pavements).

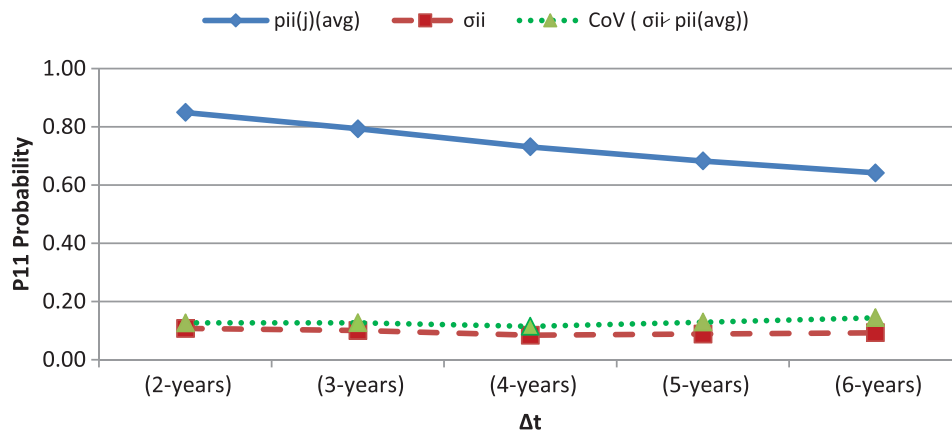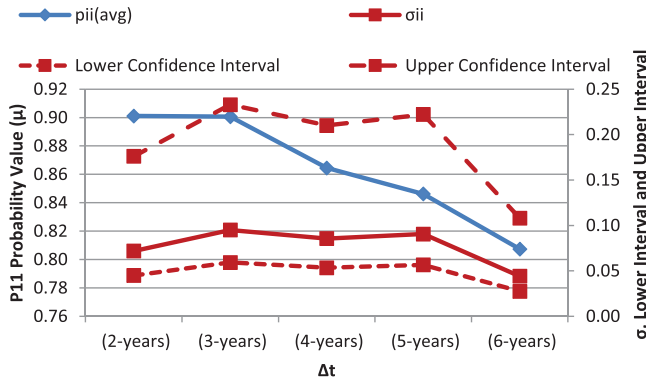| Time step Δt	$p_{11}^{(j)}$(avg)	σ_{11}	CoV
(2-years)	0.8493	0.1071	0.1262
(3-years)	0.7931	0.1001	0.1262
(4-years)	0.7308	0.0837	0.1145
(5-years)	0.6825	0.0878	0.1286
(6-years)	0.6414	0.0924	0.1441

Figure 3. Evolution of statistics with time steps (rigid pavements).

Figure 4. Evolution of statistics with time steps (semi-rigid pavements).

Table 5. Mean, standard deviation and corresponding CI of p_{11} for 5 time steps (mix pavements).

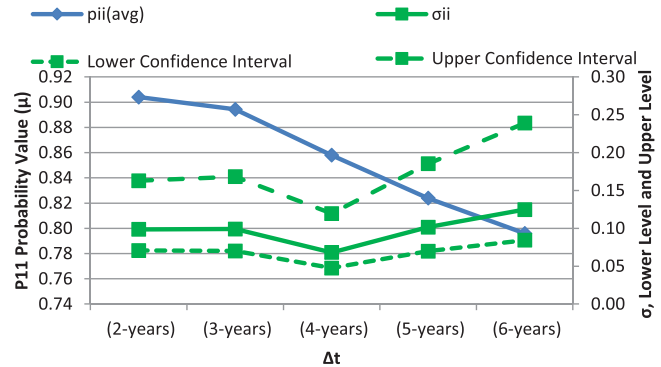| Time step (Δt)	p_{11}(avg)	σ_{11}	n_p	Lower bound of CI of σ_{11}	Upper bound of CI of σ_{11}	CI
(2-years)	0.9040	0.0986	13	0.0707	0.1628	[0.0707– 0.1628]
(3-years)	0.8942	0.0991	12	0.0702	0.1682	[0.0702– 0.1682]
(4-years)	0.8579	0.0681	11	0.0476	0.1195	[0.0476– 0.1195]
(5-years)	0.8239	0.1015	10	0.0698	0.1852	[0.0698– 0.1852]
(6-years)	0.7960	0.1247	9	0.0843	0.2390	[0.0843– 0.2390]

to find segments in the same very good state between 1 and 6 years than between 1 and 2 years.

From Table 5 and Figure 6, it is clear that the mean value is decreasing with increasing time step. However, for the standard deviation, the trend is not constant and the range of variation seems to be significant even if a linear fitting gives the equation $\sigma_{11} = 0.0055\,\Delta t + 0.082$. From Figure 6 the trend line representing the standard deviation points is still between the lower and upper limits and the projection of the standard deviation value at 4 years time step is equal to $0.005*0.0681 + 0.082 = 0.0823$, which is included in the confidence interval of [0.0476, 0.1195]. Accordingly, from the available data, the scatter of the TP can be considered as independent of time and for mix pavements, we consider σ as constant with value: 0.0823. This standard deviation is close to the one obtained for semi-rigid pavement and the property is the same. The standard deviation of the degradation process of semi-rigid and mix pavements is shown to be stable with time when it was decreasing for rigid ones.

4.4. Discussion

We were looking in this section on the evolution of the probability to stay in very good condition with the time step (P_{11}). That gives indirect information of the initiation of the ageing

Figure 6. Evolution of statistics with time steps (mix pavements).

process: the less P_{11}, the quicker is the change towards lower condition states. It was shown for all the type of pavements that this probability was decreasing significantly from about 0.9–0.6–0.8 from 2 to 6 years time step, showing a clear medium term degradation process. The effect was greater for rigid pavements (0.9–0.6) than for others (0.9–0.8); that means that rigid pavement show a lower aptitude to stay in very good conditions. The second objective was to look for an invariant in terms of scatter in the evaluation of P_{11}, if it exists. Here also it was shown that the standard deviation was higher for rigid pavements (0.1) than for semi-rigid or mix ones (0.8). Moreover, CoV of rigid pavement probability of transition
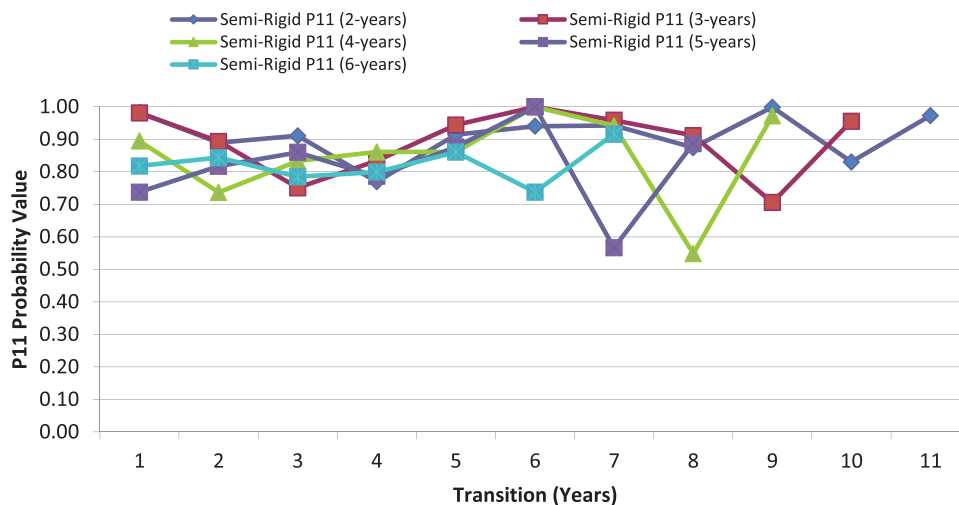

Figure 5. Evolution of p_{11} with segments age for 5 time steps (semi-rigid pavements).

was constant when the standard deviation was constant for semi-rigid or mix pavements. The first conclusion is that the degradation process of rigid pavements is more uncertain than the one of the others or that the assessment is more uncertain. The second lesson is that the probabilistic modelling of rigid pavements doesn't follow the same principle than the others.

5. Uncertainty of evaluation and its propagation for condition index assessment

Quality of uncertainty propagation is a fundamental property of a probabilistic models and was discussed and stated as a criterion in Schoefs (2008). We first model the uncertainty of evaluation for p_{ii}, called error of evaluation, by a random variable. We compute this error for rigid pavements only for which the statistical uncertainty is less. The errors of evaluation for all the p_{ii} are gathered in the same sample; that allows increasing the size of the sample: 52 values for 2 years (13 years *4 TP; $p_{11}, p_{22}, p_{33}, p_{44}$) and 36 values for 6 years (9 years * 4 TP). That helps for better modelling the overall uncertainty, especially the distribution tails. The disadvantage is that the error of evaluation for each p_{ii} cannot be deduced. To achieve this goal, each realisation of the normalised error (ε), corresponding to the jth value of the realisation of p_{ii}, is calculated using the following Equation (3)

$$\text{Normalized Error } (\varepsilon^{(j)}) = \frac{p_{ii}^{(j)} - p_{ii}(\text{avg})}{\sigma_{ii}}. \tag{3}$$

The PDFs (probability density function) of this error for 2 and 6 years time steps are plotted respectively on Figure 7 left and right. We first observe that they are symmetrical. That is explained by the fact that the assessment of p_{ii} combines several types of uncertainties (reporting, human error, human experience) that doesn't lead to a more probable under or over-estimation. Second the distribution for the 6 years time step is more tighten, showing more values around the null error. That means that the estimation is less scattered in that case;

that meets the meaning that the estimation is easier after 6 years. The shape shows that we capture almost all the values between -3σ and $+3\sigma$. Thus, the propagation of p_{ii} condition for rigid pavements is considered using three statistical estimates; mean $p_{ii}(\text{avg})$, $p_{ii}(\text{avg})-3\sigma_{ii}$ (lower boundary), and $p_{ii}(\text{avg}) + 3\sigma_{ii}$ (upper boundary).

The following equations were used where j is the time step:

$$p_{ii}^{(j)}(\text{upper}) = p_{ii}^{(j)}(\text{avg}) + 3\sigma_{ii}^{(j)}, \tag{4}$$

$$p_{ii}^{(j)}(\text{lower}) = p_{ii}^{(j)}(\text{avg}) - 3\sigma_{ii}^{(j)}. \tag{5}$$

The following matrices illustrate the TPMs for mean, lower and upper limits values for 2 and 6 years respectively. For 2 years time step, $p_{ii}(\text{avg}) = 0.8586, 0.7580, 0.6396$, and 0.4689 for p_{11}, p_{22}, p_{33} and p_{44}, respectively. $\Sigma_{ii}^{(2)} = 0.1071, 0.1952, 0.2702$, and 0.4132 for p_{11}, p_{22}, p_{33} and p_{44}, respectively. The minimum probability value is considered zero and the maximum is 1.

$$P_{(L)}^{(2)} = \begin{bmatrix} 0.5373 & 0.4283 & 0.0327 & 0.0016 & 0 \\ 0 & 0.1724 & 0.8276 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_{(U)}^{(2)} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

For 6 years time step, $p_{ii}(\text{avg}) = 0.6414, 0.5323, 0.5661$, and 0.4341 for p_{11}, p_{22}, p_{33} and p_{44}, respectively. $\Sigma_{ii}^{(6)} = 0.0924, 0.2289, 0.2215$, and 0.3503 for p_{11}, p_{22}, p_{33}, and p_{44}, respectively. The minimum probability value is
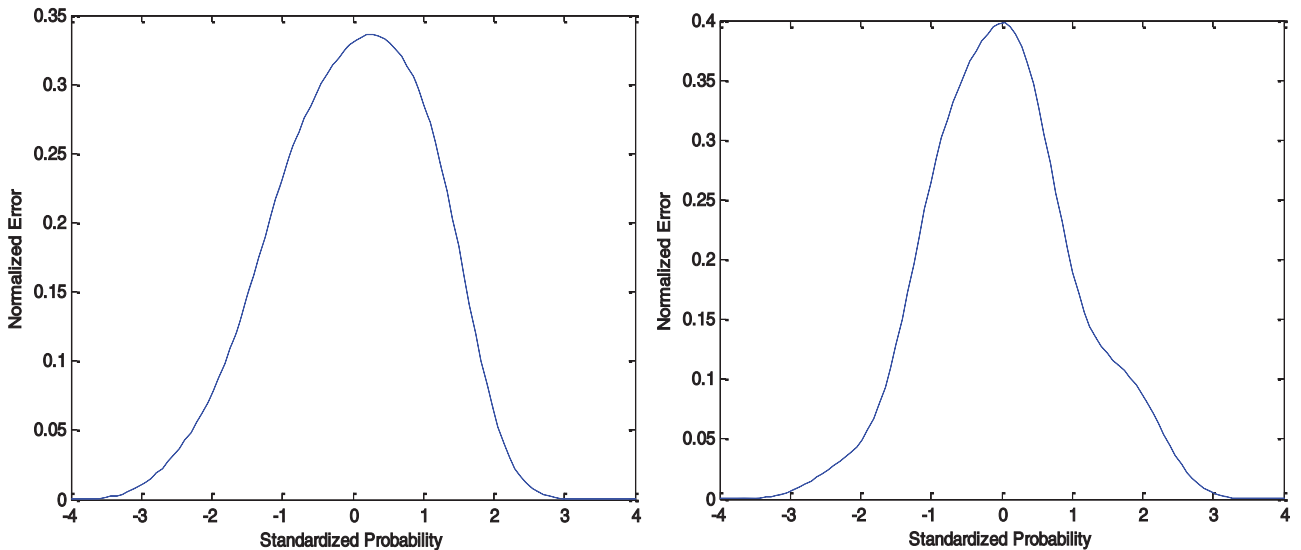

Figure 7. PDF of random variable ε (left: 2 years and right: 6 years time steps) – rigid pavements.

considered zero and the maximum is 1.

$$P_{(L)}^{(6)} = \begin{bmatrix} 0.3642 & 0.4805 & 0.1189 & 0.0364 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$P_{(U)}^{(6)} = \begin{bmatrix} 0.9186 & 0.0615 & 0.0152 & 0.0047 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

The propagation of the rigid pavements is calculated up to 30 years. The following assumptions were used:

- Initial state vector = $C_{(0)}$ = [1, 0, 0, 0, 0], where the numbers are the probabilities of having condition rating of VG, G, F, P, and VP at age 0, respectively.
- Then, the estimated condition rating at year t by MC is $C(t) = C_{(0)} \times P^t \times R'$.

Table 6 illustrates the average values for the PCI up to 30 years for 2 and 6 years time steps using first-order MC computations.

Figure 8 illustrates the comparison between the uncertainty propagation of the two time steps 2 and 6 years.

It appears clearly from this figure that the range of uncertainty for 6 years time step is less than that for 2 years. This result is completely compatible with the previous results in the paper (Figure 7). The quality of assessment of pavement condition is better defined in 6 years comparing with 2 years. The degradation process and the severity of defects which are usually resulted from the variation of traffic loads, climatic conditions, poor in original structural design, maintenance works, etc, are clearer in 6 years time step, and consequently the decision towards the appropriate maintenance policy or action is usually the suitable one. Moreover, from financial point of view, it is more realistic to perform the inventory and inspection process using medium time steps (5 for instance). Note that for semi-rigid and mix pavements a statistical uncertainty
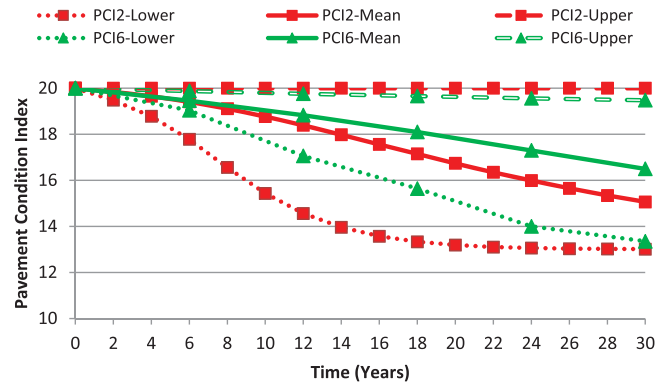

Figure 8. Uncertainty propagation of the two time steps, 2 and 6 years (rigid pavements).

is added (section 4). The present database doesn't allow studying larger time steps than 6 years with a convenient uncertainty. We know that preventive maintenance is a compromise between frequency (and cost) of inspections and uncertainty of inspection, efficiency and cost of preventive and curative maintenances. It is shown in previous cited studies about pavements maintenance and more generally for infrastructures (Bastidas-Arteaga and Schoefs 2015, Sheils *et al.* 2012) or networks (Breysse *et al.* 2007) with service lifetime usually between 50 and 70 years, that optimal inspection period for preventive maintenance is between 4 and 10 years. The proposition of this paper (around 6 years) thus respects two criteria: reduce the uncertainty of assessment and be short enough. Even if this conclusion appears reasonable, the effect of the uncertainty of assessment should be quantified in a complete maintenance optimisation.

6. Validation of the probabilistic modelling for condition state assessment

For a stakeholder, the key requirement of a probabilistic model of degradation is to be reliable (see Section 5) and simulate accurately the evolution of the condition states (error of modelling). We select her the accuracy of fitting the curve of the real cumulative distribution function (CDF) of condition states: very good, good, fair, poor and very poor, taking values respectively from 1 to 5 (Table 1). Here also, the case of rigid pavements is selected. The CDF curve for the real (observed) data and the two MC models (2 and 6 years time steps) after 6 and 12 years are presented, respectively, in Figures 9 and 10. The curves of the MC models present the mean, lower, and upper boundaries according to the uncertainties obtained in Section 5. From Figure 9, it is interesting to observe that the trend of the MC model fits well the real data for the two time steps when the pavement is in good or fair condition, which means that we can apply the preventive maintenance as the uncertainty is also controlled. However, for worse condition states, the trend of the CDF for real data is changed and the difference between the observed and MC models is increasing (uncertainty is increasing) till all curves are collected at the absorbing point: the very poor condition. In Figure 10, the same conclusion is obtained up to the good or fair condition and the trend is changed after till reaching the absorbing

Table 6. Propagation of PCI for 2 and 6 years time steps.

| Time (years)	2 Years time step			6 Years time step		
	PCI-Lower	PCI-Mean	PCI-Upper	PCI-Lower	PCI-Mean	PCI-Upper
0	20	20	20	20	20	20
2	19.48	19.84	20			
4	18.58	19.65	20			
6	17.06	19.41	20	19.04	19.46	19.88
8	15.36	19.11	20			
10	14.30	18.77	20			
12	13.70	18.39	20	17.06	18.83	19.76
14	13.38	17.98	20			
16	13.20	17.56	20			
18	13.11	17.15	20	15.64	18.10	19.66
20	13.06	16.74	20			
22	13.03	16.35	20			
24	13.01	15.99	20	14.00	17.30	19.56
26	13.01	15.65	20			
28	13.00	15.34	20			
30	13.00	15.06	20	13.35	16.50	19.48
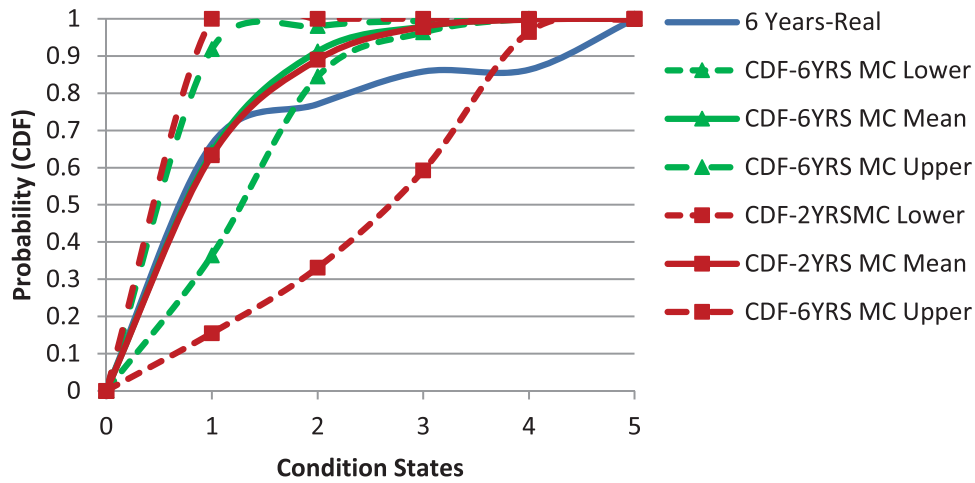
Figure 9. Cumulative distribution function for real database, 2 and 6 years MC at 6 years time period – rigid pavements.
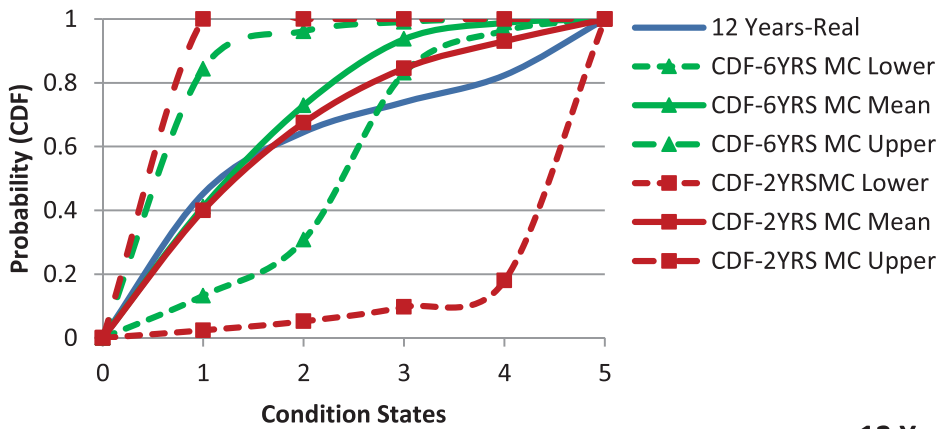
12 Years

Figure 10. Cumulative distribution function for real database, 2 and 6 years MC at 12 years time period – rigid pavements.

state with higher uncertainty range comparing with Figure 9. Accordingly, the range of uncertainty is increasing with time due to the variation in traffic loads, climatic and environmental conditions, performing of non-registered maintenance works and the MC model. More-over the estimation of the trend is bad due to the few available observations (9 segments in poor conditions for mix pavements). Even if our database seems to be massive (around 1000 data for each type), the distribution of these data in some condition is small. With four condition states, the possible data for computing p_{ii} is the total number divided by 4, but is much less for segments in poor or fair conditions.

The validation of the trend of the MC models compared with the real data profile is performed by using mean square error (MSE) in addition to the use of chi-square test for the goodness of fit. Table 7 illustrates the MSE and Chi-square test results based on 95% confidence interval and 2 degrees of

freedom ($\chi^2 \leq 3.84$) as we are interested in preventive maintenance which is corresponding to the condition states (1–3: very good-fair).

Consequently, when planning the optimal time of the maintenance strategy, the decision makers should balance between using the preventive maintenance with less indirect user cost (AitMokhtar *et al.* 2017) represented by the additional travel time and larger range of uncertainties from one side, and the long-term maintenance strategy represented by the corrective maintenance which will contribute in increasing the indirect user cost (additional travel time) but with lesser uncertainty from the other side. Finally, the MC model can be used to predict the future condition of an asset for preventive maintenance strategy and as discussed before, it should be preferred to evaluation TP with inspection time periods around 6 years. For strategy based on larger time between inspections, such as corrective maintenance, another complex model might be used such as second-order MC for example.

Table 7. Validation of MC model using MSE and chi-square test.

Model	6 Years		12 Years	
	MSE	χ^2	MSE	χ^2
Real data CDF and 2 years MC	0.0082	0.0992	0.0053	0.0651
Real data CDF and 6 years MC	0.0093	0.0096	0.0606	0.3880

7. Conclusions

In our paper the first-order MC probabilistic approach was used for pavement degradation modelling. This work is based

on the post-treatment of a survey of networks with age between 1 and 15 years. An example of Markov property verification was introduced using STSs. The MC was identified though the calculation of the TPM for different time steps and three pavement types: namely rigid, semi-rigid and mix. Two uncertainties were modelled: the uncertainty of evaluating TPM's and the error of MC modelling. The first one was evaluated and modelled for two time steps (2 and 6 years) while the second was assessed from the propagation of uncertainty in comparison with real evolution after 6 and 12 years. Accordingly, based on the previous discussion and analysis, the following points can be concluded for modelling:

- The mean value of TPM's is decreasing with increasing of time step.
- The CoV is suitable used to model the uncertainty of evaluation of rigid pavements TPMs with time step, as the standard deviation is decreasing with time step.
- The TPMs of the semi-rigid and mix pavements follow a different behaviour except the mean value: it is decreasing with increasing time step, which meets the physical meaning that the probability of the very good condition state of pavement section is decreasing with the increasing of time step. Standard deviations are shown to be constant and very close for these two types of pavements: 0.0718 for semi-rigid pavements and 0.0823 for mix pavements.
- The error of modelling is very fair for pavements in very good, good and fair condition states, if the statistical error is small (rigid pavements in the paper).

Based on the previous discussion and analysis in Section 6, it is concluded that:

- Increasing time step will contribute in decreasing the uncertainty of evaluation. Accordingly, to capture uncertainty in traffic volume, change in climatic and weather conditions, defects in pavement surface, etc., it is recommended to use 6 years time step TPM instead of 2 years to predict the future rigid PCI especially for preventive maintenance policy. The decision maker should balance between getting precise information and bigger uncertainty in case of short time step (2 years for example) or getting less precise modelling and less uncertainty in case of large time step (6 years for example).
- The propagation of uncertainty helps in identifying the proper time step, the effect of the uncertainty of assessment and showing the error of modelling. It results that the first-order MC model is suitable for preventive maintenance policy, when the degradation doesn't reach poor or very poor condition states.
- Values of TPMs and their uncertainty are provided. That gives the opportunity for comparison with other networks and to analyse the effect on preventive maintenance optimisation with a risk analysis. Even if a database seems to be massive (1000 data), the distribution of these data in condition states should be analysed. With four condition states for instance, the possible data for computing p_{ii} is the total number divided by 4.

Disclosure statement

No potential conflict of interest was reported by the authors.

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