Chinese Journal of Physics 64 (2020) 9–17

Contents lists available at ScienceDirect

Chinese Journal of Physics

journal homepage: www.elsevier.com/locate/cjph

Impurity effects on the magnetization and magnetic susceptibility of an electron confined in a quantum ring under the presence of an external magnetic field

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A B S T R A C T

The magnetization and the magnetic susceptibility of a single electron confined in a two-dimensional (2D) parabolic quantum ring under the effect of external uniform magnetic field and in the presence of an acceptor impurity have been studied. The shifted $1/N$ expansion method was used to solve the Hamiltonian quantum ring within the effective mass approximation. The computed energy spectra, the magnetization and magnetic susceptibility have been displayed as a function of the quantum ring parameters: confinement strength ω_0, magnetic field strength (ω_c), and temperature (T). The obtained energy results show level-crossings, in the absence and presence of acceptor impurity, which are manifested as oscillations in the magnetization and magnetic susceptibility curves.

1. Introduction

Fabrication methods, such as lithography techniques and molecular beam-epitaxy, have been utilized to manufacture a new nanostructure which is called quantum ring (QR) that confines carriers in its inside [1]. The study of quantum rings has received great attention due to its different technological applications in single-photon emitters, photonic detectors, nanoflash memories and qubits for spintronic quantum computing [2]. Recently, the electronic, magnetic, optical, and thermal properties of quantum ring have been investigated. The electronic states of the carriers which are confined in the quantum ring can strongly be modified by using external magnetic field [3-15]. This important subject had received intensive research. For example, in Ref. [3], Chakraborty et al. had studied the electronic states of a single quantum ring in the presence of an applied magnetic field. Impurities and geometrical effects on the electron energy spectrum had been shown by Farais et al. In Refs. [7,8,12] the authors have investigated the energy levels, magnetization and magnetic moment of the nanoring. Fomin et al. [7] have calculated the energy spectra and showed the oscillatory magnetization of the two-electron quantum ring. The authors of Refs. [6-8] have studied the absorption spectra of confined electron in a quantum ring and showed the effects of electric field and the impurity .The impurities also play an essential role in changing the properties of semiconductor materials [16-22]. Doping quantum ring with donor and acceptor impurities influences the electron quantum confinement because the ionised impurities include an additional repulsive or attractive electrostatic potential in the Hamiltonian of the quantum ring.

There are many important effects that play a major role in changing the magnetic and thermodynamic properties of nanomaterials with various reduced dimensions: quantum wells (2D), quantum well wires (1D), quantum dots (0D) and quantum rings (QRs) [23-30]. The applied fields, impurities and quantum confinement are examples of the most important factors.

Different approaches had been used to solve the quantum nanosystem's Hamiltonian, including the effect of an applied magnetic field to obtain the eigenenergies and eigenstates and to study the properties of the system [28-31]. In Ref. [30] Elsaid et al. had used

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https://doi.org/10.1016/j.cjph.2020.01.002
Received 4 October 2019; Received in revised form 31 December 2019; Accepted 9 January 2020
Available online 16 January 2020

the 1/N expansion technique to study the effect of dimensionality on the spin-singlet energy splitting in the ground state energy of a quantum dot in the presence of a magnetic. Li and Xi have studied the electronic structure of a hydrogenic acceptor impurity in semiconductor nano-structures in the framework of effective-mass envelope-function theory [31] .An analytical expression for the magnetic moment as a function of the magnetic field flux through the one-dimensional quantum ring is obtained. This expression has the oscillation characters [32]. In addition, another external factors that can modify the magnetic and thermodynamic properties of the quantum dots are: intense laser and magnetic fields, Rashba and Dresselhaus spin orbit interaction effects, topological defect,temperature, pressure and eccentricity effects [32-46]. Khordad et al. [33], very recently, have studied theoretically the thermodynamic properties of 1D quantum wire under the effect of Rashba spin–orbit interaction and magnetic field [33-34]. Baghdasaryan et al. calculated the thermal and magnetic properties of electron states in a quantum dot in an external magnetic field [35]. Sharma et al. had obtained the donor ground state binding energy and magnetic susceptibility of impurities in the quantum dot in the presence of an applied magnetic field using numerical diagonalization [47].

Xie Wen-Fang had calculated the energy level states of two-electron quantum ring under the influence of a perpendicular magnetic field using the exact diagonalization method [14-15]. Voskoboynikov had computed the energy states and magnetization in nanoscale quantum ring and found that the magnetization curve shows oscillator behavior [37]. Schwarz et al. had presented experimental studies of the magnetization of electrons in semiconductor quantum dot. The magnetization behavior shows pronounced oscillations of temperature of 0.3 K [48]. Kleemans et al. [49] have reported measurement of the persistent current carried by a single electron by means of magnetization experiments on self-assembled InAs/GaAs quantum ring. The experimental curve of the magnetic moment shows oscillator behavior.

The shifted 1/N expansion method had been implemented to solve the Hamiltonian of an electron confined in a quantum ring in the presence of acceptor impurity and a magnetic field. The obtained energy spectrum is used to calculate the statistical energies and magnetic properties as function of external magnetic field strength, parabolic confinement, and temperature in the presence of impurity. In this work, we investigate the combined effects of the magnetic field, impurities and temperature on the behavior of the magnetization and magnetic susceptibility of an electron confined in a quantum ring. Our ring is made from GaAs/AlGaAs nanomaterial with fixed ring radii and various parabolic confinement strengths. The rest of the paper is organized as follows: in Section 2, we present the Hamiltonian theory of a dopant impurity in an external magnetic field, method of computation and magnetic properties of quantum ring. The results and discussion are given in Section 3. The final section is devoted for conclusion.

2. Theory

This section displays the QR impurity Hamiltonian under the effect of an applied magnetic field, the method of computation, and the QR magnetic properties.

2.1. The quantum ring hamiltonian

A system of one electron with effective mass m^* and charge e, moving in a two-dimensional (2D) parabolic quantum ring under the effects of an external uniform magnetic field along the z-direction and confinement potential, in addition to a center impurity, can be modeled by the QR Hamiltonian as shown:

$$\hat{H} = \frac{\hbar}{2m^*}\left(\vec{\mathbf{P}} + \frac{e}{c}\vec{\mathbf{A}}\right)^2 + \frac{1}{2}m^*{\omega_0}^2(r - R_0)^2 + \frac{1}{2}g^*\mu_B B\sigma_z + \frac{\lambda\, e^2}{\epsilon\, r} \tag{1}$$

$\vec{\mathbf{P}}$ refers to the electron momentum operator which corresponds to the electron position coordinate $\vec{\mathbf{r}}\ (x, y)$. $\vec{\mathbf{A}}$ is the magnetic vector potential which is related to the applied magnetic field $\vec{\mathbf{B}} = \nabla \times \vec{\mathbf{A}}$. The vector potential is chosen to be in the symmetric gage as $\vec{\mathbf{A}} = \frac{B}{2}$ **(-y, x, 0)**, where $\vec{\mathbf{B}}$ is taken to be along z-direction and normal to the two-dimensional QR plane. ϵ is the dielectric constant of GaAs material, c is the speed of light. $\omega_c = \frac{eB}{m^*}$, is the cyclotron frequency, ω_0 is the strength of the confinement potential frequency and R_0 the average radius of the ring. The parameter lambda with values: $\lambda = 1$ *and* $\lambda = 0$, indicates the presence and absence of impurity, respectively.

The radial potential that describes the confinement of the electron in the quantum nano-ring is given by, $V(r) = \frac{a_1}{r^2} + a_2 r^2 - V_0$, where the first term represents the repulsive term, the second one is a harmonic oscillator type potential constrains the electron to the ring and $V_0 = 2\sqrt{a_1.\, a_2}$. The radial potential has a minimum at $r = R_0 = \sqrt[4]{\frac{a_1}{a_2}}$, with R_0 defining the average radius of the ring. The radial potential, $V(r)$, for r near r_0, has a simple potential parabolic form: $V_p(r) \cong \frac{1}{2}m^*\omega_0^2(r - R_0)^2$, where $\omega_0 = \sqrt[2]{\frac{8a_2}{m^*}}$ and m^* is the effective electron mass.

The confining potential, $V_p(r)$, despite its simplicity, it was shown that such a model explains experimental data, in particular, the beating effect in the oscillations pattern in the persistent current in a GaAs/AlGaAs single loop [49-50].

Then, the Hamiltonian of the system Eq. (1) can be shown as:

$$\hat{H} = \frac{-\hbar^2}{2m^*}\nabla^2 + \frac{1}{2}m^*\omega_0^2(r - R_0)^2 + \frac{1}{8}m^*\omega_c^2 r^2 + \frac{1}{2}\hbar\,\omega_c\,(L_z + gS) + \frac{\lambda\, e^2}{\epsilon\, r} \tag{2}$$

Where, L_z is the orbital angular momentum for the electron along the z-direction with eigenvalues $m_l\hbar$, where $m_l = 0, \pm1, \pm2$ is the magnetic quantum number.

2.2. Method of computation

The shifted 1/N expansion method have been applied, as an efficient computational technique, to solve the effective mass Hamiltonian with spherically symmetric potential described by Eq. (2) to compute the energies which are essential input data to calculate the statistical average energies, in order to find the magnetic properties.

The technique is vital, and it gives accurate results [51]. The starting point is to write the radial Schrodinger equation in N spatial dimensions as:

$$\left[-\left(\frac{d^2}{dr^2}+\frac{N-1}{r}\frac{d}{dr}\right)+\frac{l(l+N-2)}{r^2}+V(r)\right]\phi(r)=E\phi(r) \tag{3}$$

Where,

$$V(r)=\frac{1}{2}m^*\omega_0^2(r-r_0)^2+\frac{1}{8}\mathrm{m}^*\omega_c^2r^2+\frac{1}{2}\omega_c\,(\mathrm{m_l}+gS)+\frac{\lambda\, e^2}{\varepsilon r} \tag{4}$$

$l(l+N-2)$ is the eigenvalue of the square of the orbital angular momentum operator in N dimension space and $l=|m_l|$.

The main point of the shifted $1/N$ expansion method is to rewrite Eq. (3) by using a parameter k, $(k=N+2l)$, and a suitable shift parameter a.

Now, Eq. (3) can be rewritten in terms of the shifted variable, $\bar{k}=k-a$ as:

$$\left[-\frac{d^2}{dr^2}+\bar{k}^2\left(\frac{[1-(1-a)/\bar{k}][1-(3-a)/\bar{k}]}{4r^2}+\frac{V(r)}{Q}\right)\right]\phi(r)=\phi(r) \tag{5}$$

Where Q is a scaling constant to be determined later. For large $\bar{k}$, the main contribution to the energy E comes from the effective potential (V_{eff}), so the kinetic energy becomes negligible

$$V_{eff}=\frac{1}{4r^2}+\frac{V(r)}{Q} \tag{6}$$

The minimum r_0 of V_{eff} is given by the relationship:

$$2r_0^3V'(r_0)=Q \tag{7}$$

The origin of coordinates should be shifted to the position of the minimum (r_0) of the effective potential by defining a new variable x.

$$x=\frac{\bar{k}^{1/2}}{r_0}(r-r_0) \tag{8}$$

An analytical equation similar to the Schrodinger equation of the one-dimensional solvable anharmonic oscillator can be obtained by expanding Eq. (3) around r_0 (respectively $x=0$). Then the coefficients of both equations can be compared to define all the anharmonic oscillator parameters in terms of $\bar{k}$, Q, r_0 and the potential derivatives in order to obtain the energy spectrum as in references [51-53].

For fixed values of QR- physical parameters:ω_0, ω_c,R_0,λ,n_r (radial quantum number) and for any value of m_l, the energy $E(n_r,m_l)$ expression for the state $|n_r,\, m_l\rangle$, in powers of $1/\bar{k}$ (up to third order) is given as:

$$E=E_0+E_1+E_2+E_3 \tag{9}$$

Where E_0,E_1, E_2, E_3 are energy terms where the complete expressions are given in terms of QR-parameters, quantum numbers and potential derivatives, in the reference [51-53].

The shift parameter a is chosen to make E_1 vanishes. This condition produces the exact energy result for the Hydrogen atom case, so,

$$a=2-(2n_r+1)\omega \tag{10}$$

ω is the anharmonic oscillator frequency

$$\omega=\left[3+\frac{r_0V''(r_0)}{V'(r_0)}\right]^{1/2} \tag{11}$$

An equation for determining the root (r_0) is given below:

$$(2r_0^3V'(r_0))^{1/2}=2+2l-a\;\omega \tag{12}$$

With this value of r_0, we can determine ω, a and every identified parameter, which completes all necessary steps to compute the energy eigenvalues of $V(r)$.

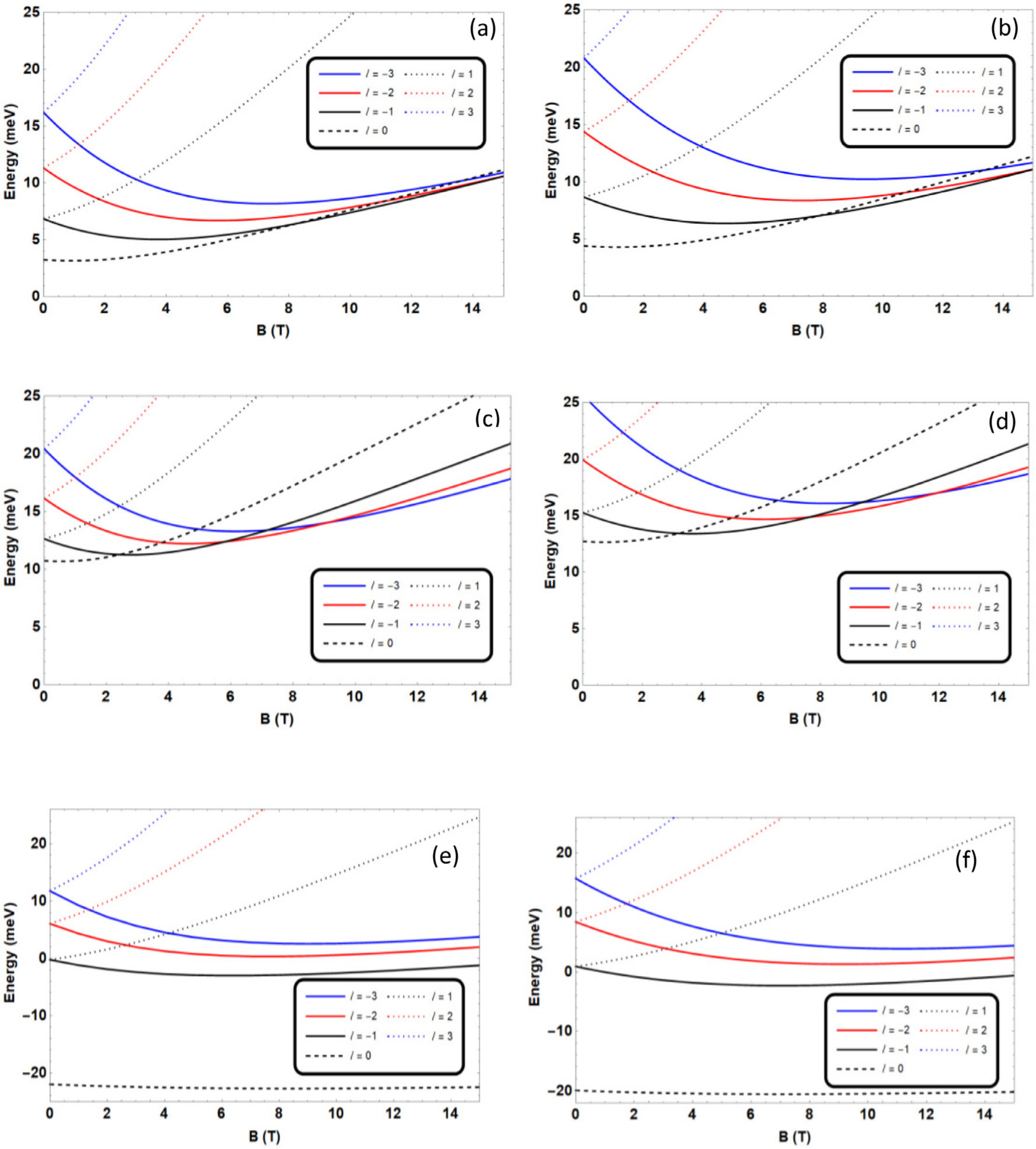

Fig. 1. Energy spectrum of the QR as function of magnetic field for $\lambda = 0$ in (a and b), $\lambda = +1$ in (c and d), $\lambda = -1$ in (e and f), for two values of confinement strengths $\omega_0 = 7$ meV in the left panel and $\omega_0 = 10$ meV in the right panel, dashed lines for ($n = 0,\ l = 0$), solid lines for ($n = 0,\ l = -1, -2, -3$) and the dotted lines for ($n = 0,\ l = +1, +2, +3$), all l values are labeled from below at $B = 0$.

2.3. The magnetic properties of the quantum dot

The Magnetization (M) can be computed by differentiating the average statistical energy of the 2D QR system with respect to the magnetic field strength B.

$$M(T,\, \omega_0,\, B,\, \lambda) = -\frac{\partial \langle E \rangle}{\partial \mathrm{B}} \tag{13}$$

Where,

$$\langle E \rangle = \frac{\sum_{\alpha=1}^{N_{max}} \mathrm{E}_\alpha e^{-\mathrm{E}_\alpha(\mathrm{B})/k_B T}}{\sum_{\alpha=1}^{N_{max}} e^{-\mathrm{E}_\alpha(\mathrm{B})/k_B T}} \tag{14}$$

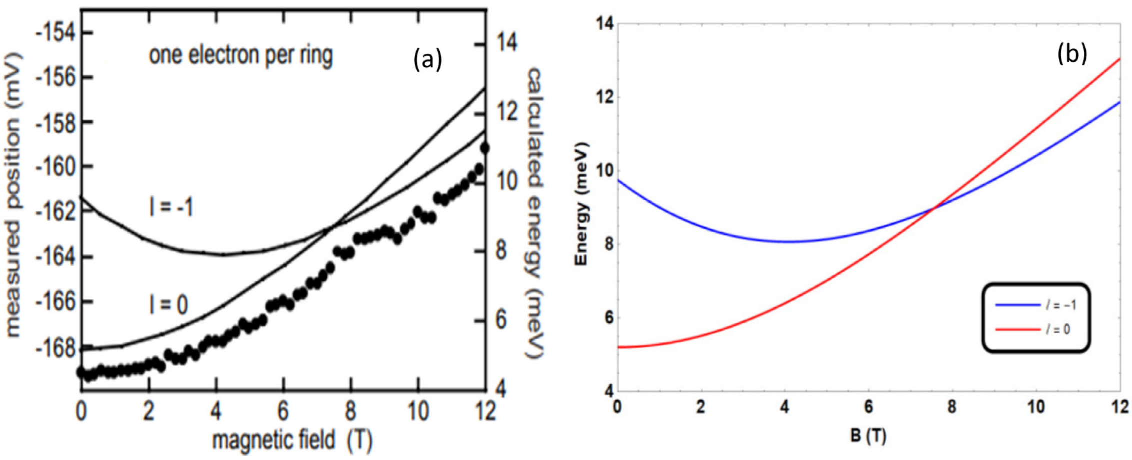

Fig. 2. Energy spectrum of the QR as function of magnetic field a) taken from Ref. [1], dotted line for experimental data, and solid line for calculated results b) present work, the parameters in two figures are ($R_0 = 14$ nm, $\hbar\omega_o = 12$ meV).

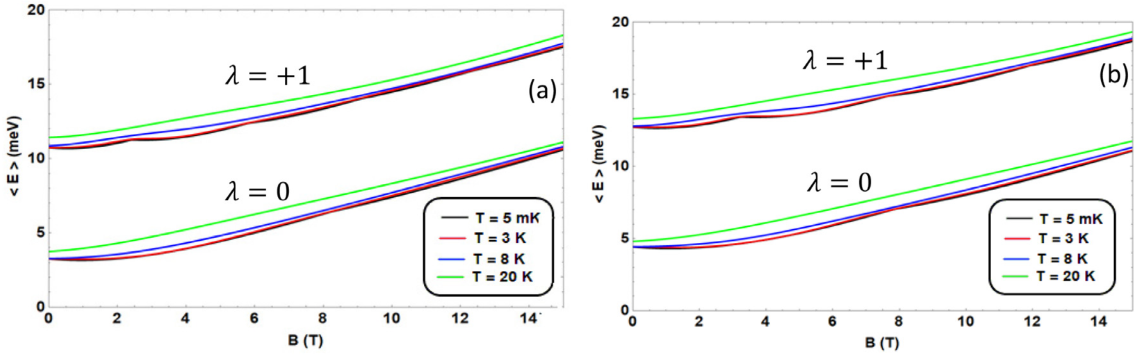

Fig. 3. Average statistical energy vs magnetic field for absence and presence of impurity for $\omega_0 = 7$ meV in (a) and $\omega_0 = 10$ meV in (b) for different temperature.

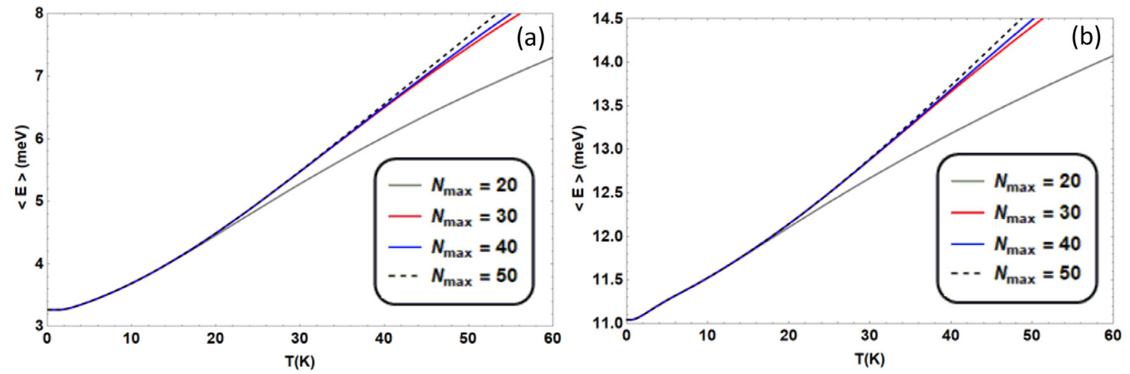

Fig. 4. Statistical average energy functions of temperature for different N_{max} (a) without impurity (b) with acceptor impurity for $\omega_0 = 7$ meV and $B = 2$T, using $N_{max} = 20$, 30, 40, *and* 50, labeled from below at $T = 45$ K.

α represents the occupied QR –state. By substituting Eq. (14) in Eq. (13), the magnetization (M) can be expressed in a common standard from as:

$$M(T,\,\omega_0,\,B,\,\lambda) = -\frac{\sum_{\alpha=1}^{N_{max}} \frac{\partial \mathrm{E}_\alpha(\mathrm{B})}{\partial B}\, e^{-E_\alpha(\mathrm{B})/k_B T}}{\sum_{\alpha=1}^{N_{max}} e^{-E_\alpha(\mathrm{B})/k_B T}} \tag{15}$$

The magnetic susceptibility (χ), in the presence of the donor impurity, can be found by differentiating the magnetization (M) with respect to the magnetic field strength B:

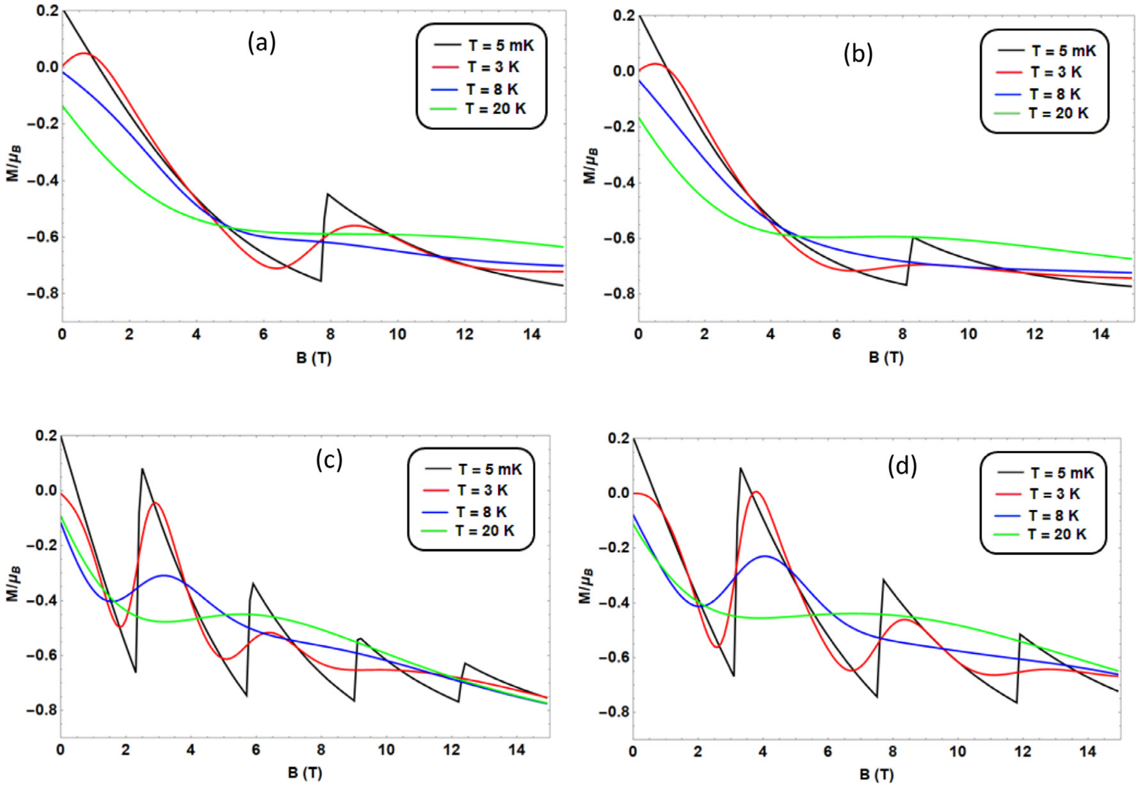

Fig. 5. Magnetization vs magnetic field for $\lambda = 0$ in (a and b), $\lambda = +1$ in (c and d), For two value of confinement strength $\omega_0 = 7$ meV in the left panel and $\omega_0 = 10$ meV in the right panel and different temperature.

$$\chi = \frac{\partial M}{\partial B} \tag{16}$$

3. Results and discussion

In this work, we have used the GaAs material parameters: $m^* = 0.067m_0$ and $\epsilon = 12.4$ and fixed ring radius 14 nm. In Fig. 1a and b, we have shown the energy spectra of the QR as a function of magnetic field in the absence of impurity, ($\lambda = 0$), for two confinement strengths $\omega_0 = 7$ and 10 meV. The figures show the intersections between the states and the change in the angular momentum of the ground state. Fig. 1c and d present the QR energies under the effect of the acceptor impurity ($\lambda = 1$) as a function of the magnetic field. For $l<0$ the energy reveals a minimum at certain value of magnetic field and this minimum becomes deeper and shifts to right as $|l|$ is increasing. The intersection between the energy corresponding to negative l and the energy corresponding to $l = 0$ can be noticed clearly from Fig. 1c and d which causes the change of ground state angular momentum. For $\omega_0 = 7$ meV, the acceptor impurity spectra shows an intersection of $l = -1$ and $l = 0$ at $B \approx 2.5$ T and another intersection of $l = -2$ and $l = -1$ at $B \approx 6$ T. This energy level crossing,which appeared as a cusp in the energy –magnetic field curve, had been reported experimentally for quantum ring of radius $R_0 = 14\ nm$ and confinement energy $\hbar\omega_0 = 12meV$, as shown in Fig. 2a in Ref. [1]. The comparison between the present theoretical results computed by 1/N expansion method, displayed in Fig. 2b, against the experimental and theoretical results, plotted in Fig. 2a, shows a very good agreement. This interesting phenomenon of the energy level crossing significantly appears in the magnetic properties which shall be presented later. By comparing the ($\lambda = 0$ and $\lambda = +1$) QR-spectra plots, we can observe that by adding the acceptor impurity the no-impurity energy states shifted up to higher values. As a result of this, the change in the angular momentum of the ground state occurs at lower magnetic field values. In addition, we can notice that by increasing the confinement strength (ω_0), the level crossing moves to right (to higher magnetic field). In Fig. 1e and f we have studied the effects of donor impurity ($\lambda = -1$) on the QR-energy spectra, for example, the energy state ($l = 0$) is shifted down to a negative energy, this behavior removes the energy level crossing.

In Fig. 3a and b the average statistical energy as a function of the magnetic field strength have been displayed for two values of confinement frequency. We have also studied the effect of the acceptor impurity on the statistical energy curve. On one side, the figures show a concave in the curve of the energy as increasing in the magnetic field specially at low temperatures ($T = 5$ mK and $T = 3$ K). On the other side, for higher temperatures, the energy increases without significant behavior. The effect of acceptor impurity can be summarized by increasing the values of the statistical energy curves and making the cusps more evident.

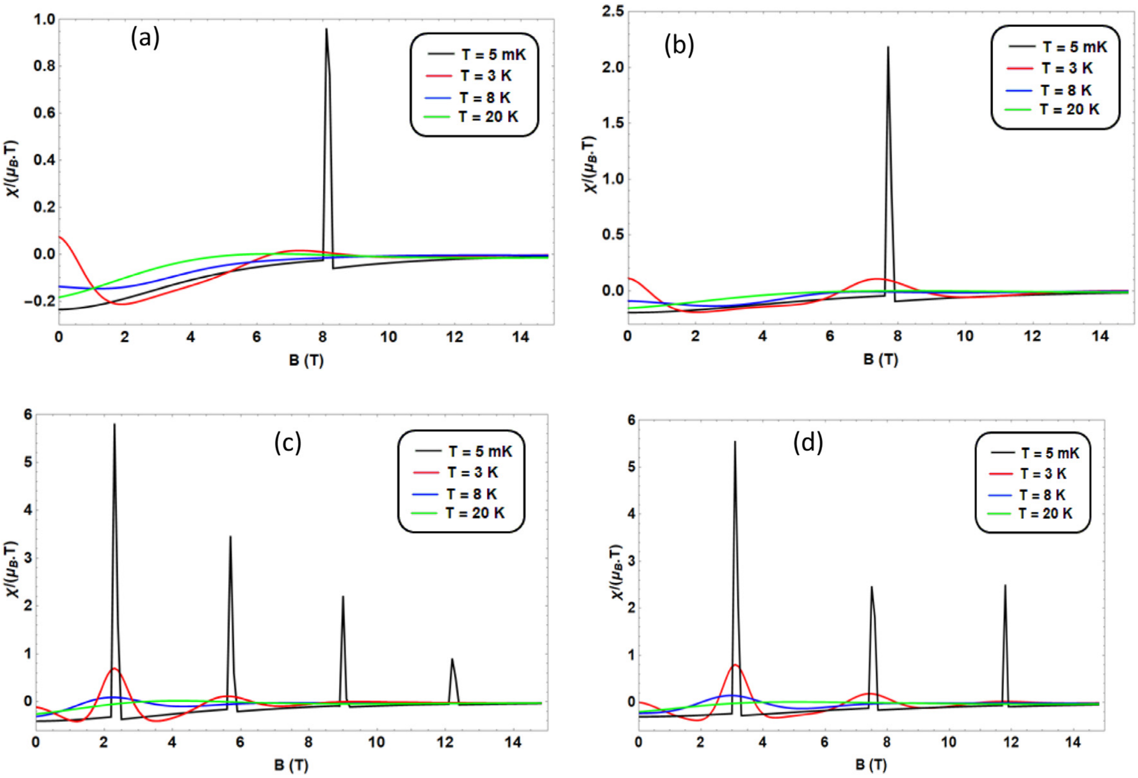

Fig. 6. Magnetic susceptibility vs magnetic field for $\lambda = 0$ in (a and b), $\lambda = +1$ in (c and d), For two values of confinement strength $\omega_0 = 7$ meV in the left panel and $\omega_0 = 10$ meV in the right panel and different temperature.

To ensure the convergence issue, we have plotted in Fig. (4) the average statistical energy as a function of temperature for $\omega_0 = 7$ meV and $B = 2$ T, with different number of bases ($N_{\max}$), for $\lambda = 0$ and $\lambda = +1$. We found that $N_{\max} = 50$ is enough to give accurate statistical calculations for both cases; the presence and absence of the acceptor impurity. The temperature range $T \leq 20$ K is used in the present calculations.

In Fig. 5, we investigate the dependence of the magnetization on magnetic field and temperature for no-impurity case (Fig 5 (a and b)) and acceptor impurity (Fig. 54 (c and d)), for two different confinement frequencies. It is clear from Fig. 5 that there are cusps in the magnetization curves due to energy level crossing. By increasing the temperature, we can notice that the oscillating behavior gradually disappears, and the curves become smoother. Moreover, the effect of the acceptor impurity is to increase the number of cusps in the magnetization curves. On one hand, the peaks in the magnetization curves are shifted to higher values of the magnetic field for the higher value of the confinement strength $\omega_0 = 10$ meV. On the other hand, this behavior can be observed obviously in the case of presence of the acceptor impurity, while no significant changes are observed for no-impurity case. The significant change in the magnetization curve due to the change in the confinement strength can be attributed to the competition between confinement and coulomb interaction terms in the QR Hamiltonian. For fixed values of magnetic field, by decreasing ω_0, electron-impurity repulsion becomes more effective and this makes the electron unstable, so the electron jumps to a higher angular momentum state in order to reduce its coulomb energy.

Now, QR magnetic susceptibility as function of magnetic field graphs are presented in Fig. 6. For the no-impurity case, we can observe only one peak in Fig. 6 (a and b) which takes place at $B \approx 8\,T$ for $\omega_0 = 7$ meV and 10 meV. The peak in the magnetic susceptibility curve corresponds to the energy levels crossing (as in Fig. 1 (a and b)). However, for the acceptor impurity case three peaks in the magnetic susceptibility curve can be noticed clearly as shown in Fig. 6 (c and d) at ($B \approx 2.2, 5.8, 9,$ *and* $12.2\,T$) in c and $B \approx 3.1, 7.5, 11.7\,T$ in d for $\omega_0 = 7$ meV and 10 meV, which match the level crossing in (Fig. (1)(c and d)) . The first peak corresponds to the transition from $l = 0$ *to* $l = -1$ of the ground state angular momentum. Besides, the second one is from $l = -1$ *to* $l = -2$ and so on.

Finally, magnetic susceptibility dependence on the temperature has been investigated in Fig. 6 (a, b, c, and d). We can conclude that by increasing the temperature the peaks become wider and smoother.

4. Conclusion

In conclusion, we have solved the Hamiltonian of single electron, moving in a two-dimensional 2D parabolic quantum ring under the effects of an external uniform magnetic field and temperature, in addition to a center impurity. The shifted $1/N$ expansion method has been applied to solve the Hamiltonian and to study the spectra, statistical energies, magnetization and the magnetic susceptibility quantities of the system. In this work, we have concentrated on the level crossing in the energy states which corresponds to transitions in the angular momentum of the ground state. We have also deduced from our results that these transitions are the cause of the cusps in the magnetization and the magnetic susceptibility curves. In addition, it has been observed that the acceptor impurity significantly affects the magnetic susceptibility of the QR. Moreover, the behaviors of the magnetization and the magnetic susceptibility curves (in presence and absence of the impurity cases) were shown as function of the magnetic field strength (B), temperature (T) and confinement strength (ω_0). The computed results show the effects of the system's parameters on the behavior of the peaks in the magnetization and the magnetic susceptibility curves.

Declaration of Competing Interest

The Author declare there is no conflict of interest.

Acknowledgments

This research receives funding from An-Najah National University Grant No. ANNU-1819-Sc021

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