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Research article

Education

IMPACT OF THE MATHEMATICAL MODELING ON CONCEPTUAL UNDERSTANDING AMONG STUDENT-TEACHERS

数学模型对学生-教师概念理解的影响

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Abstract

The present study investigates the effect of mathematical modeling on conceptual understanding among student-teachers. Also, the study proposes relevant materials and activities in mathematical modeling. Two classes of 140 student-teachers participated in the study. Mathematical modeling instruction was used in the treatment group, while the comparison group was taught by a teacher-centered method. T-Test results showed a difference between the treatment and comparison groups in terms of conceptual understanding. Moreover, the mathematical modeling group showed improvement in the knowledge, comprehension, and application of geometry and measurement concepts. The results of this study are novel as they are introduced to student-teachers to facilitate teaching mathematics among children. The researchers suggested engaging student-teachers in rich modeling activities, which would deepen their students' mathematics learning.

Keywords: Mathematical Modeling, Concepts, Understanding, Student-Teachers

摘要 本研究调查了数学建模对师生概念理解的影响。此外，该研究还提出了数学建模中的相关材料和活动。两个班级的 140 名师生参加了这项研究。治疗组采用数学建模教学，对照组采用以教师为中心的教学方法。吨检验结果显示治疗组和对照组在概念理解方面存在差异。此外，数学建模小组在几何学和测量概念的知识、理解和应用方面都取得了进步。这项研究的结果是新颖的，因为它们被介绍给学生教师，以促进儿童数学教学。研究人员建议让师生参与丰富的建模活动，这将加深学生的数学学习。

关键词: 数学建模、概念、理解、师生

I. INTRODUCTION

The need to build understanding and promote critical thinking among learners is constantly increasing due to the enormous flow of daily experiences in learners' lives as well as the significant and genuine desire to enable learners to become knowledge creators rather than just knowledge consumers.

Thus, educational systems in many countries have begun to seek educational policies that are capable of empowering learners and integrating them into active learning environments [1]. These policies enable students to feel that they are self-learners, increase their enthusiasm, enhance their cognitive and practical skills, give them opportunities to be responsible for their learning, and encourage their individual approaches to acquiring, employing, and generating knowledge [2].

The mathematics curriculum is considered fertile ground for interpreting the educational objectives related to the implementation of learners' thinking in various real-life applications. In fact, analyzing, decision making, planning, and building economic and administrative models are no longer possible without a mathematical structure of functional dimensions [3]. Parallel to these, a study [4] recommended revising the mathematics curriculum and updating professional development programs for mathematics teachers to enrich their mathematical knowledge and prepare them for the task of creating interactive classes.

Mathematical modeling is a bridge through which the teacher can facilitate students' learning of mathematics. Modeling represents mathematical concepts and presents them in drawing or in embodiment, linking them to the reality of learners and their daily lives. Modeling also contributes to the development of understanding and thinking. Moreover, mathematical modeling and its applications and skills are necessary for mathematics teachers to introduce something new to learners [5]. According to Freudenthal [6], mathematical modeling is a tool of creativity and is a simple and clear idea that improves mathematical knowledge and provides meaning to the learning process. It is a method of mathematizing life events, plans, and pedagogical components. In the same context, some researchers [7] believe that mathematical modeling is a process of simulating real-world problems in mathematical terms and finding solutions to these problems using a mathematical model that can be dealt with in a simple way compared to the complexity of the problems in the real world. In other words, mathematical modeling aims to transform a real-world problem into a mathematical problem, solve the latter, and translate the solution into realistic conditions. Greefrath and Vorhölter [8] agreed with [7] in presenting mathematical modeling as a process of developing a model based on real-life problems and using the model to solve the identified problems.

Ramirez-Velarde [9] pointed out that when teachers use models, students gain a deeper understanding of the relevant knowledge and acquire problem-solving skills through the learning experiences of making strong connections among mathematical representations, symbols, and concrete materials. Therefore, the modeling process includes the definition and identification of the characteristics of the suitability of real-world experiences, the representation of these characteristics with symbols, the analysis and interpretation of the model and the characteristics of the experience, and the consideration of accuracy within the limits of the model [10].

In mathematical modeling, language also plays a very important role because it is a resource that supports mathematical symbolism, notations, and images in the process of building mathematical meanings and ideas [11]. In its standards for school mathematics, one study [10] argued that the purposes of using mathematical modeling are to deepen the relationships among different topics of mathematics (e.g., numbers, algebra, and geometry) and to solve problems faced by learners due to the impact of such a process on the understanding of mathematics. The use of mathematical modeling is also believed to be a vital step in developing learners' understanding of mathematics, improving their ability to solve mathematical issues, and giving them opportunities to enjoy mathematics, which in turn, may increase their self-confidence and improve their mathematical skills [12].

Furthermore, mathematical modeling instruction is in line with the principles of constructivism theory, through which learners build their understanding of the subject in general, contemplate learning, practice manual work, create models, deepen their understanding of mathematical concepts, and enhance their ability to solve mathematical problems [13]. In her study, English [14] recommended the activation of mathematical modeling in solving mathematical problems and pointed out the importance of allowing learners themselves to create models rather than limiting them to simply using existing models.

In their study, Schuchajlow and colleagues [15] found that the mathematical modeling strategy had a positive effect on learners' thinking, apart from shifting the focus of education to learners rather than teachers who used to lead the solution before using modeling to solve mathematical questions. Another study [14] concluded that learners' critical thinking skills improved while building mathematical models, which also influenced their ability to work independently. Furthermore, this process developed their aesthetic sense, as they were given opportunities to enjoy mathematics by studying mathematical models and geometric shapes.

For the reasons stated above, there is a need to apply mathematical modeling and measure its impact on learners' understanding of mathematical concepts. The approach of the current study matches the mathematical modeling approach proposed by [16]. This approach is based on the idea of engaging students in teacher-supervised modeling tasks in situations in which the students are assisted in acquiring new and useful mathematical concepts and knowledge.

Accordingly, the current study aims to measure the impact of mathematical modeling on the understanding of the mathematical concepts of student teachers at the Faculty of Educational Sciences of the An-Najah National University.

A. Research Problem

Learning and teaching mathematics and the use of mathematical models may cast a shadow on learners' understanding. Furner and Worrell [17] believed that understanding mathematical concepts helps learners' make correct connections between the materials and the concepts presented.

The researchers noted that the teaching of mathematics not only involves helping students understand concepts and build vital skills, but also enabling them to receive knowledge of mathematics for the purposes of examinations and measuring achievements. Student teachers use regular materials in courses related to teaching children mathematics. They also use school textbooks and teach theoretical concepts in mathematics classes. However, there are no advanced and constructive methods for investing in learners' abilities. It is also worth noting that the current student teachers are going to teach children mathematics, and the majority of them graduated with a human sciences background from secondary schools. Their background and knowledge of mathematics are not very strong, so mathematical modeling could enhance their mathematical knowledge and concepts and make them appreciate its value in teaching mathematics.

Based on the above, the study tries to answer the following main questions:

1. What is the impact of mathematical modeling on the Faculty of Educational Sciences learners' knowledge of the mathematical concepts?
2. What is the impact of mathematical modeling on Faculty of Educational Sciences learners' comprehension of the mathematical concepts?
3. What is the impact of mathematical modeling on applying mathematical concepts among the Faculty of Educational Sciences learners?

B. The Study Contribution

This study aims to identify the mathematical modeling strategy's effect on knowing, understanding, and applying mathematical concepts among the Faculty of Educational Sciences students.

The practical importance of this study lies in its ability to provide mathematics teachers with learning strategies. Mathematical modeling can contribute to stimulating learners and learning mathematics in an environment that is rich in understanding.

This study is novel as its theoretical importance of understanding mathematical concepts will be shown as a mathematics structure basis since it is considered a reliable indicator of understanding mathematical construction as a whole. It also provides a theoretical framework for mathematical modeling and its use in learning and teaching mathematics.

This paper is novel because it seeks to contribute to the current debate in the literature due to mathematical modeling. Math educators presented mathematical modeling as an instructional strategy and a series of activities. The scientific novelty of the research also consists of a conducted large-scale study on conceptual understanding.

C. The Theoretical Framework of the Study

Mathematical modeling is essentially a mathematical application. It transforms a real-life problem into a mathematical one, solving it with lifestyle solutions and choosing the best solutions. Kahn and Kyle [18] defined mathematical modeling as translating a problem from the real world into a mathematical representation and then solving the issue according to a mathematical formula. The mathematical solution is then translated into the context of life and reality. Moreover, [19] saw mathematical modeling as the process of representing real-world problems mathematically and then finding solutions to those problems. [20] stated that mathematical modeling is an interactive process used to understand the problem or situation in different scientific fields or any other area of human knowledge. English [21] concluded that students should be required to construct mathematically significant and meaningful models that describe, explain, or predict a system.

Grandgenett et al. [22] defined modeling as a mathematical process involving observation of the phenomenon, guessing relationships, applying mathematical analyses (e.g., equations, algebraic expressions), reaching mathematical results, and reinterpreting the model. The mathematical model attempts to describe the mathematical relationships of a set of problems or situations while it is refined, purified, and tested.

Bilgic and Uzil [23] pointed out that mathematical modeling uses real-world situations to address a question that arises from outside mathematics, then moves to mathematical methods to shed light on the real-world question. This integrated modeling process proceeds from the problem's origin to building and using the model being tested to make predictions. The mathematical model is a category of assumptions and relations that are employed to solve real-life problems.

King [24] believed that mathematical modeling is a process of representing (simulating) real-world problems according to mathematical conditions and finding solutions to these problems using a mathematical model that can be dealt with more simply than the complexity of the problem in the real world. In other words, mathematical modeling transforms a real-world problem into a mathematical problem, solves that problem, and then translates the solution into real conditions.

The modeling involves defining and determining the properties of the suitability of the real-world situation, representing these characteristics with symbols, analyzing and interpreting the model and the characteristics of the situation while considering the accuracy of the model boundaries. For example, students in the middle stage must use linear equations to model a wide range of phenomena and discover several nonlinear phenomena, while secondary students study modeling more deeply. For instance, the generation or use of data and the discovery of pairing that closely match those data [10], Figure 1 illustrates the concept of mathematical modeling.

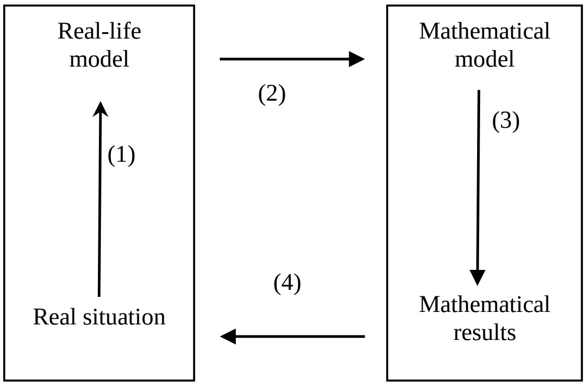

Figure 1. Explaining the concept of mathematical modeling adapted from [16] and [40]

Mathematical modeling and models in Figure 1 were related to socio-cultural perspective as [25] presented models and mathematical modeling as a significant social factor of learning mathematics. According to Rosa and Orey [26], mathematical modeling and models have become important tools to describe and solve problems within cultural, economic, political, social, and environmental systems, and further increase advantages to mathematics learning. D'Ambrosio [27] discussed the frame of mathematical models and viewed them as a translation and understanding of real-world situations and cultural contexts. Within the same perspective, Araújo [28] described models and mathematical modeling as main sources for the restructuring of reality through meaningful projects and activities. Zawojewski and colleagues [29] argued that mathematical modeling activities should be used within model development sequences, where learners work in groups of three to four. These groups could work on creating shareable models in addition to real-life problems, which can improve interaction among the students. They may hold meetings to discuss their models and rethink using them in their society. The researchers believe that presenting real-life situations or problems to the learners and asking them to discuss, design, and apply a mathematical model to solve the problems is the heart of mathematical modeling. The steps of beginning with real problems, making different mathematical models, and solving them using a group designed model, formulate a socio-cultural interaction among learners and indicate how mathematics strongly shapes the world.

Jiang [30] illustrated that mathematical modeling is a topic that concerns different areas and plays a crucial role in developing better outcomes or predicting the transformation of current items in the future. This is to improve day-to-day life and employ mathematical methods in the expression, organization of information, and the creation of graphs and pieces of evidence. It also involves solving problems from all branches of science, advanced fields, and technology. Thus, mathematical modeling plays a vital role in providing students with basic and important knowledge in mathematics and other fields in addition to developing logical thinking processes and learning how to learn [43]. It also trains learners to formulate good survey questions that lead them to research solutions to problems using evidence and persuasive arguments, which helps them increase self-confidence and gain independence in their learning. There are many reasons to teach mathematics from realistic situations familiar to students to strengthen their motivation, especially when they know that what they learn can be applied to life [42].

The National Council of Teachers of Mathematics in the United States of America refers to the importance of mathematical modeling as a process to achieve the goals of learning algebra and promote the representation of mathematical concepts [10].

Blum et al. [31] noted that mathematical modeling is of particular importance and significance in the school math curriculum. Mathematical modeling and problem solving are the appropriate ways to develop general comprehension and trends. Mathematical modeling helps develop students' critical proficiency, i.e., their ability to observe and judge independently (know, understand, analyze, define examples of mathematics, conclude, and suggest solutions to important social issues).

D. Mathematical Modeling and Teaching Concepts

To improve the knowledge, comprehension, and application of mathematical concepts, [25] introduced five principles in teaching using mathematical modeling. These five principles are as follows:

1) Model Construction

Learners construct an explicit description, clarification or process for a mathematically significant situation through a real life problem.

2) Generalizability

Learners suggest solutions that are shareable with others and similar to related situations. Teachers guide learners to develop a model for the current situation and learners generalize their model for other related situations.

3) Model Documentation

Learners create some form of documentation to demonstrate their thinking about the problem. Through this documentation, learners present and describe their developed model and production process.

4) Reality

Learners can form meaningful interpretations from their different levels of mathematical skills and general knowledge in realistic contexts.

5) Self-Assessment

Learners identify, test, and revise their current ways of thinking. Learners assess how their model satisfies and solves real-life problems; for example, through feedback from other group members and other groups.

The researchers believe that mathematical modeling instruction possesses specific characteristics and features that help the learner to actively understand and build mathematical concepts through which the learner represents the mathematical concept in a real life context, which, in itself, offers the learner strength and depth of understanding.

[21] conducted a longitudinal three-year study on data modeling in which first-grade students and teachers engaged in activities in science classes. The activities aimed to represent characteristics and phenomena from the children's environment with graphic models that compare differences and characteristics, interpret differences, and make predictions and projections. The results of the study indicate that students' develop data sense, as well as focus on what data means, understand the difference between characteristics, and improve predictability. The study has also recommended that modeling be actively deployed in school mathematics.

Moreover, [15] conducted a study in Germany with 224 students in the ninth grade. Students were asked about their level of enjoyment. The values and interests that arose when modeling mathematical subjects were discussed verbally within a purely mathematical context. The results indicated that mathematical modeling had a positive effect on students and shifted the focus of education to the learner rather than the teacher who led the solution before using modeling to solve mathematical problems. The researchers recommended greater support for learners to provide more scaffolding in their modeling of learning mathematics [15].

English [32] conducted a study on mathematical modeling in the basic stage by observing the work of students in the sixth grade to create a consumer guide to support consumer modeling with the help of their teachers, in order to develop a sense of mathematical operations and the work of a consumer-guided mathematical model.

The activities were designed as problems requiring solutions for consumers after a number of mathematical treatments. It was noted that students modified the model provided to consumers owing to their development of reflection and critical skills, as well as the inclusion of other elements and processes in the model. It was also noted that students' critical thinking skills had increased while building the mathematical model, along with their ability to work independently [32].

Carrejo [33] conducted a study to investigate the relationship between mathematical modeling and motion theory by integrating the charts into learning mathematics. Twenty-three physics and mathematics teachers participated in the study and learned the unit of movement for five days (six hours per day). The results indicated that using charts for mathematical modeling helped to illustrate several concepts for mathematics and physics teachers, such as intermediate speed. The results also provided a "good model" for predicting movement and added a study of the concepts of movement and its generalizations.

In 2003, [34] proposed a comparison between cross-curricular approaches to mathematical modeling and studied the outcomes of learning modeling and the structure of models in pre-algebra. The study included a program of five activities and was presented to two schools. In the first school, the activities contained several models that illustrate a specific problem. In the second school, the same problems were presented with a specific focus on open-ended modeling. The study found that the performance of students who learned through modeling to solve open-ended problems was better than other students.

Conceptual understanding means a comprehensive and connected understanding of mathematical concepts, ideas and generalizations. When students learn mathematics based on conceptual understanding, their consideration of mathematics as an integrated system depends on comprehension and persuasion. Through conceptual understanding, students build their mathematical knowledge on a solid background, which enables them to learn new concepts and facts by connecting them to what they already know and to real-life situations. Conceptual understanding supports connections since the students realize and produce several representations for the same concept [35].

Tu and Synder [36] described conceptual understanding through three main steps: literacy, reasoning, and thinking. Farrokhnia et al. [37] concluded that learners should be supported to achieve a deep conceptual understanding so that they reach the education system's goals of meaningful learning and knowledge transfer. It is crucial for student teachers to acquire conceptual understanding in mathematics in order to strengthen their mathematical knowledge and to help children to learn mathematical concepts meaningfully and deeply. Mathematical modeling may improve student teachers' conceptual understanding as they will experience mathematical concepts by using concrete, representative and abstract models.

II. RESEARCH DESIGN

The research design chosen for the current study was quasi-experimental and based on the selection of a treatment group of student-teachers and a comparison group, both pre- and a post-test. A quasi-experimental design was used to determine the effect of mathematical modeling on conceptual understanding among student-teachers. A pre-test of 20 multiple choice items comprising numerical mathematical concepts was applied for both groups for equivalence purposes. The results are shown in Table 1.

Table 1.
Means, standard deviations, and t-test of pretest of mathematical concepts scores

Group	N	Mean	S. D	D.F	T- value	P.
Comparison	70	11.77	1.44	138	0.459	0.647
Treatment	70	11.86	1.51			

Note: p < .05

Table 1 shows no significant difference between the treatment and comparison groups regarding the pretest of mathematical concepts.

A test of mathematical concepts was administered to both groups at the end of the study as a posttest. Mathematical modeling lessons and activities were planned, designed, and implemented by the researchers in the treatment group. The posttests of mathematical concepts scores of student-teachers in both groups were compared to find out the effects of mathematical modeling on conceptual understanding. The current study was conducted during a mathematics education course at the An-Najah National University in Nablus city for elementary school student-teachers in Spring 2019. The teacher of the course was the same for both groups.

A. Participants

The participants of the study were N = 140 student-teachers in the Faculty of Education. Furthermore, none of the student-teachers had any mathematical modeling experience in their previous courses. There were 70 (61 females, nine males) student-teachers in the treatment group in which mathematical modeling was implemented, and there were 70 (57 females, 13 males) student-teachers in the comparison group. The mean age for all participants is 21.2 years. All of the student-teachers voluntarily participated in the study.

B. Data Collection Instrument

The mathematical concepts test developed by the researchers was administered to determine the levels of the conceptual understanding of students in the treatment and comparison groups. The test consisted of 20 multiple choice items with two levels—knowledge and comprehension—in addition to five open questions measuring the application of mathematical concepts.

C. Reliability and Validity of the Instruments

The researchers used Cronbach's alpha to check the reliability and internal consistency of the test items. These estimated values were 0.82. The reliability values of each level were 0.73, 0.75, and 0.78. Moreover, the researchers presented the test to a group of experienced and specialized arbitrators in mathematics education to verify test validity using the content validity method [41]. The content validity reveals the extent to which items adequately represent the content or measure it. Two subject matter experts review the instruments, and three experts assess the content validity in relation to using modeling in mathematics.

D. Process

1) Instruction in the Treatment Group

The treatment group was N = 70 student–teachers divided into two heterogeneous classes. The teacher taught these student–teachers in mathematics courses by modeling geometry and measurement concepts. Each concept presented four models and the student teachers created equivalent or similar models. For example, they used the doors movement for modeling acute, right, and obtuse angles. They also pictured real-life situations for those angles. The modeling activities required the student–teachers to do several tasks. At the beginning of each activity, they were asked to write their definition or understanding of a specific concept derived from real problems. The second task focused on presenting a real-life model for that concept. In the third task, the student–teachers expressed their understanding by using a new model, applying the concept, and finding symbols for the concept or connecting two different models for the same concept. Finally, student–teachers formed communication groups, and each group presented all the different concept models. An example of this approach of mathematical modeling was introduced in a real-life problem involving the volume of an ancient olive tree. Each group of learners discussed the problem, presented a real-life model, expressed it as a mathematical model which they used to find a mathematical solution, and applied it to the real-life problem. In another example, students faced a problem in calculating the area of the stadium of a Nablus municipality. First, they drew a geometrical model of the stadium. Second, each student group discussed the model to have it modified. Third, students applied it mathematically. Finally, they applied the model in finding the area of the stadium.

The researchers applied 21 lessons based on mathematical modeling for three months, and each lesson lasted 90 minutes. Every lesson treated one modeling task. These instructional tasks and content included parallel and perpendicular lines, creating angles, geometrical shapes, finding the sum of angles, inscribed and central angles, perimeter, area, solid geometry, volume.

2) Instruction in the Comparison Group

The comparison group was N = 70 student-teachers taught by the teacher-centered method by applying the curriculum in mathematics. They taught 21 lessons within the same time (three months) as the student-teachers of the treatment group. The teacher-centered method is based on lesson plans, specific goals, mathematical contents in geometry, and measurement. The teacher introduced every single concept, and the student-teachers' role was to follow the teacher's steps in knowing mathematics.

3) Data Analysis

T-test for independent samples was conducted to analyze the effects of mathematical modeling on conceptual understanding. In the current study, mathematical modeling was the independent variable, and conceptual understanding was the dependent variable.

Examples of mathematical modeling activities: The student-teachers practiced several mathematical modeling activities such as:

The student-teachers determined the geometrical shape matching with the stadium (the real-life model). That shape was a rectangle (the mathematical model). This activity was evidence of the mathematical concept knowledge. The student-teachers drew two rectangles to present the external and the internal areas of the stadium. This activity is evidence of comprehension of mathematical concepts. Then the student-teachers discussed the most representative mathematical model to calculate the stadium area; they tested each model and found the area in every model. This activity was evidence of the application of mathematical concepts. By the end, the student-teachers agreed on a mathematical model and applied it to find the real area of Nablus stadium. This series of activities helped the student-teachers to understand the concept of area, estimating height and width, as well as differentiating between internal and external rectangle areas and between the two rectangles. Figure 2 presents the mathematical modeling of the area of the stadium.

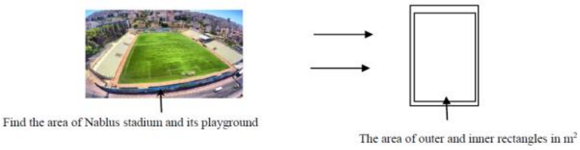

Figure 2. Modeling of the area of the stadium of Nablus municipality

The student-teachers simulated the ancient olive tree (real life model) as a cylinder (mathematical model). This activity enabled them to recognize the mathematical concept needed for calculations. They designed sketches of cylinder height and radius in order to understand the volume of a cylinder. After several calculations and estimations, the student-teachers chose a primitive model and applied it to calculate the volume of the olive tree. The student-teachers explained which factors hindered them and which factors facilitated their ability to form a mathematical model similar enough to the olive tree. Figure 3 presents the mathematical modeling of the volume of the ancient olive tree.

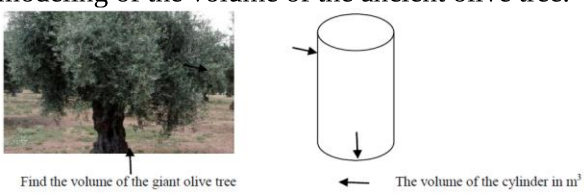

Figure 3. Modeling the volume of the ancient olive tree

III. Results

A. Knowledge of Mathematical Concepts

First hypothesis: There are no significant differences in the knowledge of mathematical concepts due to mathematical modeling.

The descriptive statistics and t-test results of the knowledge of mathematical concepts test are given in Table 2.

Table 2.
Means, standard deviations, and t-test of knowledge of mathematical concepts scores

Group	N	Mean	S. D	D.F	T-value	P.
Comparison	70	5.41	4.52	138	2.388	0.018
Treatment	70	7.01	3.32			

Note: $p < .05$

Table 2 shows a significant difference between the treatment and comparison groups in terms of their knowledge of mathematical concepts in favor of the treatment group, $r^2 = 0.04$. Moreover, the effect size was small. The findings indicate that mathematical modeling in the treatment group was more effective in improving the knowledge of mathematical concepts of students' teachers than the teaching in the comparison group.

B. Comprehension of Mathematical Concepts

Second hypothesis: There are no significant differences in the comprehension of mathematical concepts due to mathematical modeling.

The descriptive statistics and t-test results of the comprehension of mathematical concepts test are given in Table 3.

Table 3.
Means, standard deviations, and t-test of comprehension of mathematical concepts scores

Group	N	Mean	S.D	D.F	T-value	P
Comparison	70	5.33	4.17	138	2.238	0.027
Treatment	70	6.69	2.90			

Note: $p < .05$

Table 3 shows a significant difference between the treatment and comparison groups regarding their comprehension of mathematical concepts in favor of the treatment group, $r^2 = 0.035$. Moreover, the effect size was small. The findings indicate that mathematical modeling in the treatment group was more effective in improving the comprehension of mathematical concepts of student-teachers than the teaching in the comparison group.

C. Application of Mathematical Concepts

Third hypothesis: There are no significant differences in the application of mathematical concepts due to mathematical modeling.

The descriptive statistics and t-test results of applying the mathematical concepts test are given in Table 4.

Table 4.
Means, standard deviations, and t-test of applying mathematical concepts scores

Group	N	Mean	S. D	D.F	T-value	P
Comparison	70	3.01	2.35	138	3.242	0.001
Treatment	70	4.20	1.96			

Note: $p < .05$

Table 4 shows a significant difference between the treatment and comparison groups in terms of applying mathematical concepts in favor of the treatment group, $r^2 = 0.071$. Moreover, the effect size was medium. The findings indicate that mathematical modeling in the treatment group was more effective in improving the application of mathematical concepts of student-teachers than the teaching in the comparison group.

IV. Discussion and Implications

The high impact of the mathematical modeling on the participants' understanding of mathematical concepts can be traced back to several reasons, such as the fact that student-teachers created concepts through representing shapes, e.g., making angles and triangles by using hard papers, building modeling, and manual work, which improved the opportunities for understanding and deepening mathematical concepts as the sum of the pentagon angles.

Mathematical modeling also provided a good opportunity for students to employ more of their senses in building mathematical concepts. These opportunities were highlighted when they found the connections between symbols, drawing, and the model, which led to the interaction between the two parts of the brain and increased the quality and quantity of the process of learning concepts. An example of these connections was presented through an activity that linked the law of cuboid volume and its construction. Moreover, the mathematical modeling strategy allowed the exchange of dialogue and discussion, asking questions regarding the differences and similarities between the cube and the cuboid and presenting sensory evidence in support of code-based conclusions. This may have illustrated a number of concepts that were abstract mathematical ideas. Keller [38] argued that mathematical modeling provides multiple modes of understanding and builds interaction between mathematics and its learners. Hardman [39] indicated that mathematical concepts embody theoretical knowledge, which could be expressed as symbolic tools in the form of models.

Moreover, the mathematical modeling raises the level of inquiry into mathematical concepts, which has intrigued students regarding the discovery of subsequent steps in their construction of the concepts, which was familiar and clear to them before understanding such concepts unlike the teacher-centered method.

The results of this study are consistent with the results of other studies [15, 21, 33, 34] on the effectiveness of mathematical modeling in the understanding of the content of mathematical concepts and in increasing the academic results accordingly.

The researchers think that the high impact of mathematical modeling in recalling mathematical concepts may be due to the representation of mathematical concepts in models, shapes, and maps. Most mathematical modeling situations require mathematical illustrations such as drawing a perimeter with a 2D shape, a net with a 3-D shape, and a graph of a geometrical word problem such that they are based on the translation of mathematical ideas into images and models as a sign of translation of concepts. The entire mathematical situation is drawn as an understanding, helping to stabilize and store mathematical information and linking it to the mathematical models that have been used.

The researchers also believe that drawing sketches and suggesting several picture alternatives facilitate the application of mathematical concepts and their representation in models. All this results from mathematical modeling based on understanding. The transformation of mathematical elements from case to case has helped a better understanding by transforming the abstract image into a drawing or a model, which deepened the learners' understanding of geometry and measurement concepts. The learners became more aware and reflective in their local context. They discovered environmental objects that could use as models to represent mathematical concepts. In addition, mathematical modeling offered the student-teachers an educational environment full of suspense, excitement, understanding, and reflection. It gave them the freedom to interpret mathematical experience and construct it with drawings, shapes, and patterns. This helps the development of their conceptual understanding.

The researchers believe that applying concepts through modeling has been rich in forming relations and links among concepts, generalizations, and mathematical skills. The creation of modeling links between data and the required solutions and processing information with representations and models to reach the required have helped develop the ability to think inductively through a trace beginning from the existing and known knowledge and ends with the knowledge of the unknown [40].

Based on the study results, the following recommendations can be made:

1. To teach learners according to the mathematical modeling in the teaching of mathematical concepts;
2. To include the mathematical modeling in the school mathematics curriculum;
3. To enrich the mathematics teaching methods with mathematical models constructed and designed through activities practiced by learners during class activities.

V. CONCLUSION

Mathematical modeling gives the learners solid knowledge of mathematics through meaningful modeling tasks designed by the teacher. It is also an effective means to understand mathematics and a strong frame of learning and teaching mathematics if future teachers realize the nature of mathematics. Children desire to play mathematics and with mathematics, so it is necessary to design programs for future math teachers with courses and activities based on mathematical modeling.

A. Limitations

This study results are limited by a test applied to student-teachers at the An-Najah National University. The results are also limited to mathematical modeling activities from Palestinian mathematics textbooks.

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REFERENCES

[1] ORGANISATION FOR ECONOMIC CO-OPERATION AND DEVELOPMENT (2016) *Innovating Education and Educating for Innovation: The Power of Digital Technologies and Skills*. Paris: Organisation for Economic Co-operation and Development Publishing.

[2] DARLING-HAMMOND, L., FLOOK, L., COOK-HARVEY, C., BARRON, B., and OSHER, D. (2020) Implications for educational practice of the science of learning and development. *Applied Developmental Science*, 24 (2), pp. 97-140.

[3] UNITED KINGDOM DEPARTMENT OF EDUCATION (2014) *National curriculum in England: mathematics programmes of study*. [Online] Available from: https://www.gov.uk/government/publications/national-curriculum-in-england-mathematics-programmes-of-study [Accessed 14/10/21].

[4] SPARAPANI, E., CALLEJO, D., GOULD, J., HILLMAN, S., and CLARK, L. (2014) A Global Curriculum? Understanding Teaching and Learning in the United States, Taiwan, India, and Mexico. *SAGE Open*, 4 (2), pp. 1–15.

[5] TEODORO, V. and NEVES, R. (2011) Mathematical modelling in science and mathematics education. *Computer Physics Communications*, 182, pp. 8-10.

[6] FREUDENTHAL, H. (1987) Mathematics starting and staying in reality. In: WIRSZUP, I. and STREET, R. (eds.) *Proceedings of the USCMP Conference on Mathematics Education on Development in School Mathematics around the World*. Reston, Virginia: National Council of Teachers of Mathematics.

[7] NORTON, A. (2018) Frameworks for modeling students' mathematics. *Journal of Mathematical Behavior*, 52, pp. 201-207.

[8] GREEFRATH, G. and VORHÖLTER, K. (2016) Teaching and Learning Mathematical Modelling: Approaches and Developments from German Speaking Countries. In: *Teaching and Learning Mathematical Modelling. ICME-13 Topical Surveys*. Cham: Springer, pp. 1-42.

[9] RAUL RAMIREZ-VELARDE, R., ALEXANDROV, N., PEREZ-CAZARES, R., and CARLOS BARBA-JIMENEZ, C. (2015) Mathematical Modelling Based Learning Strategy. *Procedia Computer Science*, 51 (1), pp. 1694-1704.

[10] NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS (2000) *Principles and Standards for School*

Mathematics. Reston, Virginia: National Council of Teachers of Mathematics.

[11] WILKINSON, L. (2019) Learning language and mathematics: A perspective from Linguistics and Education. *Linguistics and Education*, 49, pp. 86-95.

[12] DUNDAR, S., GOKKURT, B., and SOYLU, Y. (2012) Mathematical modelling at a glance: a theoretical study. *Procedia - Social and Behavioral Sciences*, 46, pp. 3465–3470.

[13] HENNIG, C. (2010) Mathematical Models and Reality: A Constructivist Perspective. *Foundations of Science*, 15 (1), pp. 29-48.

[14] ENGLISH, L. (2020) Teaching and learning through mathematical problem posing: commentary. *International Journal of Educational Research*, 102, 101451.

[15] SCHUKAJLOW, S., LEISS, D., PEKRUN, R., BLUM, W., MÜLLER, M., and MESSNER, R. (2012) Teaching Methods for Modelling Problems and Students' Task-specific Enjoyment, Value, Interest and Self-Efficacy Expectations. *Educational Studies in Mathematics*, 79 (2), pp. 215-237.

[16] KAISER, G. (1995) *Mathematical Modelling in School Examples & Experience*. Hamburg: Department of Education, University of Hamburg.

[17] FURNER, M. and WORRELL, L. (2017) The Importance of Using Manipulatives in Teaching Math Today. *Transformations*, 3 (1), 2.

[18] KAHN, P. and KYLE, J. (2002) *Effective Learning & Teaching Mathematics & Its Applications*. London: Kogan Page Limited.

[19] CHENG, A. (2001) Teaching mathematical modeling in Singapore school. *The Mathematics Educator - Association of Mathematics Educators*, 6 (1), pp. 63-75.

[20] ROSA, M. (2000) *From reality to mathematical modeling: A Proposal for using ethno mathematical knowledge*. Unpublished doctoral thesis, California State University.

[21] ENGLISH, L. (2012) Data Modelling with First-Grade Students. *Educational Studies in Mathematics*, 81 (1), pp. 15-30.

[22] GRANDGENETT, N.F., OSTLER, E., ZYGIELBAUM, A., HENNIGER, S., and HAZZARD, C. (2000) Mathematical modeling within a technology based learning environment: Some principles for adaptive instruction. *Proceedings of the Mathematics, Science, Education and Technology Conference, San Diego, California, February 2000.* Available from https://ncite.unl.edu/Articles/affinity/Math_Science_Education_and_Tech%20Conf_2000.pdf.

[23] BILGIC, E. and UZIL, D. (2014) The attitudes of the elementary school mathematics teacher candidates towards proof in the mathematical modelling process. *Procedia - Social and Behavioral Sciences*, 116, pp. 1083–1087.

[24] KING, A. (2004) Phylogenetic comparative analysis: a modeling approach for adaptive evolution. *The American Naturalist*, 164, pp. 683–695.

[25] LESH, R., HOOVER, M., HOLE, B., KELLY, A., and POST, T. (2000) Principles for developing thought-revealing activities for students and teachers. In: KELLY, A. and LESH, R. (eds.) *Research Design in Mathematics and Science Education*. Mahwah, New Jersey: Lawrence Erlbaum and Associates, pp. 591-646.

[26] ROSA, M. and OREY, D. (2010) Ethnomodeling as a Pedagogical Tool for the Ethnomathematics Program. *Revista Latinoamericana de Etnomatemática*, 3 (2), pp. 14-23.

[27] D'AMBROSIO, U. (2015) Mathematical modelling as a strategy for building-up systems of knowledge in different cultural environments. In: STILLMAN, G.A., BLUM, W., and BIEMBENGUT, M.S. (eds.) *Mathematical modelling in education research and practice: cultural, social and cognitive influences*. New York: Springer, pp. 35-44.

[28] ARAÚJO, J. (2009) Formatting real data in mathematical modelling projects. *IMFUFA Text*, 461, pp. 229–239.

[29] ZAWOJEWSKI, J. and LESH, R. (2003) A models and modelling perspective on problem solving. In: LESH, R.A. and DOERR, H. (eds.) *Beyond constructivism: Models and modeling perspectives on*

mathematics problem solving, learning, and teaching. Mahwah, New Jersey: Lawrence Erlbaum, pp. 317-336.
[30] JIANG, L. (2005) *Mathematical modeling and methods of option pricing*. Singapore: World Scientific Publishing.
[31] BLUM, W., GALBRAITH, P., HENN, H.W., and NISS, M. (2006) *Applications and modelling in mathematics education*. New York: Springer.
[32] ENGLISH, L. (2006) Mathematical Modeling in the Primary School: Children's Construction of a Consumer Guide. *Educational Studies in Mathematics*, 63 (3), pp. 303-323.
[33] CARREJO, D. (2004) *Mathematical Modeling and Kinematics: A Study of Emerging Themes and Their Implications for Learning Mathematics through an Inquiry-Based Approach*. PhD. thesis, The University of Texas at Austin.
[34] LEGE, G. (2003) *A Comparative case study of contrasting instructional approaches applied to the instruction of mathematical modeling*. Proquest Information and Learning Company, Education in Teachers College Columbia University, UMI, No. 3091273.
[35] EVERS, T. (2011) *Wisconsin department of public instruction*. [Online] Available from: http://dpi.wi.gov/ [Accessed 16/07/11].
[36] TU, W. and SNYDER, M. (2017) Developing conceptual understanding in a statistics course: Merrill's First Principles and real data at work. *Educational Technology Research and Development*, 65 (3), pp. 579–595.
[37] FARROKHNIA, M., PIJEIRA-DÍAZ, H.J., NOROOZI, O., and HATAMI, J. (2019) Computer-supported collaborative concept mapping: The effects of different instructional designs on conceptual understanding and knowledge co-construction. *Computers & Education*, 142, 103640.
[38] KELLER, E. (2002) *Making Sense of Life: Explaining Biological Development with Models, Metaphors, and Machines*. Cambridge, Massachusetts: Harvard University Press.
[39] HARDMAN, J. (2019) Towards a pedagogical model of teaching with ICTs for mathematics attainment in primary school: A review of studies 2008–2018. *Heliyon*, 5 (5), e01726.
[40] BLUM, W. (1996) Applications and Modelling in Mathematics Education. *Journal for Mathematics*, 23 (3), pp. 262-280.
[41] QUANSAH, F., AMOAKO, İ., and ANKOMAH, F. (2019) Teachers' Test Construction Skills in Senior High Schools in Ghana: Document Analysis. *International Journal of Assessment Tools in Education*, 6 (1), pp. 1-8.
[42] SNOOK, K. and ARNEY, D. (2002) A continuum of choice: Instructional techniques in undergraduate mathematics. In: *Proceedings of the Interdisciplinary Workshop on Core Mathematics: Considering Change in the First Two Years of Undergraduate Mathematics*. Washington, District of Columbia: Mathematical Association of America, pp. 143-149.
[43] TEZER, M. (2019) The Role of Mathematical Modeling in STEM Integration and Education. In: FOMUNYAM, K.G. (ed.) *Theorizing STEM Education in the 21st Century*. IntechOpen. Available from https://www.intechopen.com/chapters/68547.

参考文:

[1] 经济合作与发展组织（2016）创新教育和创新教育：数字技术和技能的力量。巴黎：经济合作与发展组织出版。
[2] DARLING-HAMMOND, L.、FLOOK, L.、COOK-HARVEY, C.、BARRON, B. 和 OSHER, D. (2020) 对学习和发展科学教育实践的影响。应用发育科学，24 (2)，第 97-140 页。
[3] 英国教育部 (2014) 英国国家课程：学习数学课程。[在线] 可从：https://www.gov.uk/government/publications/national-curriculum-in-england-mathematics-programmes-of-study [访问 14/10/21]。
[4] SPARAPANI, E.、CALLEJO, D.、GOULD, J.、HILLMAN, S. 和 CLARK, L. (2014) 全球课程？了解美国、台湾、印度和墨西哥的教学和学习。圣人公开赛，4 (2)，第 1-15 页。

[5] TEODORO, V. 和 NEVES, R. (2011) 科学和数学教育中的数学建模。计算机物理通信，182，第 8-10 页。

[6] FREUDENTHAL, H. (1987) 数学开始并停留在现实中。在：WIRSZUP, I. 和 STREET, R. (编辑。) USCMP 数学教育与世界各地学校数学发展会议论文集。弗吉尼亚州雷斯顿：全国数学教师委员会。

[7] NORTON, A. (2018) 学生数学建模框架。数学行为杂志，52，第 201-207 页。

[8] GREEFRATH, G. 和 VORHÖLTER, K. (2016) 数学建模教学与学习：德语国家的方法和发展。在：教学和学习数学建模。国际商会-13 专题调查。查姆：斯普林格，第 1-42 页。

[9] RAUL RAMIREZ-VELARDE, R.、ALEXANDROV, N.、PEREZ-CAZARES, R. 和 CARLOS BARBA-JIMENEZ, C. (2015) 基于数学建模的学习策略。继续计算机科学，51 (1)，第 1694-1704 页。

[10] 国家数学教师委员会 (2000) 学校数学的原则和标准。弗吉尼亚州雷斯顿：全国数学教师委员会。

[11] WILKINSON, L. (2019) 学习语言和数学：语言学和教育的视角。语言学与教育，49，第 86-95 页。

[12] DUNDAR, S.、GOKKURT, B. 和 SOYLU, Y. (2012) 数学建模概览：理论研究。继续-社会和行为科学，46，第 3465-3470 页。

[13] HENNIG, C. (2010) 数学模型与现实：建构主义视角。科学基础，15 (1)，第 29-48 页。

[14] ENGLISH, L. (2020) 通过数学问题的教学和学习：评论。国际教育研究杂志，102，101451。

[15] SCHUKAJLOW, S., LEISS, D., PEKRUN, R., BLUM, W., MÜLLER, M., 和 MESSNER, R. (2012) 建模问题和学生特定任务享受的教学方法，价值，兴趣和自我效能的期望。数学教育研究，79 (2)，第 215-237 页。

[16] KAISER, G. (1995) 学校实例和经验中的数学建模。汉堡：汉堡大学教育系。

[17] FURNER, M. 和 WORRELL, L. (2017) 今日数学教学中使用操作的重要性。转换，3 (1), 2。

[18] KAHN, P. 和 KYLE, J. (2002) 有效学习与教学数学及其应用。伦敦：高根佩奇有限公司。

[19] CHENG, A. (2001) 在新加坡学校教授数学建模。数学教育者 - 数学教育者协会，6 (1)，第 63-75 页。

[20] ROSA, M. (2000) 从现实到数学建模：使用民族数学知识的建议。未发表的博士论文，加州州立大学。

[21] ENGLISH, L. (2012) 与一年级学生的数据建模。数学教育研究，81 (1)，第 15-30 页。

[22] GRANDGENETT, N.F.、OSTLER, E.、ZYGIELBAUM, A.、HENNIGER, S. 和 HAZZARD, C. (2000) 基于技术的学习环境中的数学建模：自适应数学的一些原则。数学、科学、教育和技术会议论文集，加利福尼亚州圣地亚哥，2000 年 2 月。可从 https://ncite.unl.edu/Articles/affinity/Math_Science_Education_and_Tech%20Conf_2000.pdf 获得。

[23] BILGIC, E. 和 UZIL, D. (2014) 小学数学教师候选人在数学建模过程中对证明的态度。继续-社会和行为科学，116，第 1083-1087 页。

[24] KING, A. (2004) 系统发育比较分析：适应性进化的建模方法。美国博物学家，164，第 683-695 页。

[25] LESH, R.、HOOVER, M.、HOLE, B.、KELLY, A. 和 POST, T. (2000) 为学生和教师开展思想揭示活动的原则。在：KELLY, A. 和 LESH, R. (编辑。) 数学和科学教育研究设计。摩诃，新泽西州：劳伦斯· 埃尔鲍姆和同事，第 591-646 页。

[26] ROSA, M. 和 OREY, D. (2010) 生态模型作为民族数学项目的教学工具。拉丁美洲经济杂志，3 (2)，第 14-23 页。

[27] D’AMBROSIO, U. (2015) 数学建模作为在不同文化环境中建立知识系统的策略。在：STILLMAN, G.A., BLUM, W., 和 BIEMBENGUT, M.S. (编辑。) 教育研究和

实践中的数学建模：文化、社会和认知影响。纽约：斯普林格，第 35-44 页。
[28] ARAÚJO, J. (2009) 在数学建模项目中格式化真实数据。国际货币基金组织文本，461，第 229-239 页。
[29] ZAWOJEWSKI, J. 和 LESH, R. (2003) 问题解决的模型和建模视角。在：LESH, R.A.和 DOERR, H.（编辑）超越建构主义：数学问题解决、学习和教学的模型和建模观点。摩诃，新泽西州：劳伦斯· 埃尔鲍姆，第 317-336 页。
[30] JIANG, L. (2005) 期权定价的数学建模和方法。新加坡：世界科学出版社。
[31] BLUM, W.、GALBRAITH, P.、HENN, H.W. 和 NISS, M. (2006) 数学教育中的应用和建模。纽约：斯普林格。
[32] ENGLISH, L. (2006) 小学数学建模：儿童消费者指南的构建。数学教育研究，63 (3)，第 303-323 页。
[33] CARREJO, D. (2004) 数学建模和运动学：新兴主题及其对通过基于探究的方法学习数学的影响的研究。博士。论文，德克萨斯大学奥斯汀分校。
[34] LEGE, G. (2003) 应用于数学建模教学的对比教学方法的比较案例研究。探索信息和学习公司，哥伦比亚大学师范学院教育，乌米，编号 3091273。
[35] EVERS, T. (2011) 威斯康星州公共教育部。[在线] 可从：http://dpi.wi.gov/ [访问 16/07/11]。
[36] TU, W. 和 SNYDER, M. (2017) 在统计课程中发展概念理解：美林的第一原则和工作中的真实数据。教育技术研究与开发，65 (3)，第 579-595 页。
[37] FARROKHNIA, M.，PIJEIRA-DÍAZ, H.J., NOROOZI, O., 和 HATAMI, J. (2019) 计算机支持的协作概念映射：不同教学设计对概念理解和知识共建的影响。计算机与教育，142、103640。
[38] KELLER, E. (2002) 理解生命：用模型、隐喻和机器解释生物发展。马萨诸塞州剑桥：哈佛大学出版社。
[39] HARDMAN, J. (2019) 迈向利用信息通信技术促进小学数学学习的教学模式：2008-2018 年研究回顾。赫利永，5 (5)，e01726。
[40] BLUM, W. (1996) 数学教育中的应用和建模。数学杂志，23 (3)，第 262-280 页。
[41] QUANSAH, F.、AMOAKO, İ. 和 ANKOMAH, F. (2019) 加纳高中教师考试建设技能：文件分析。国际教育评估工具杂志，6 (1)，第 1-8 页。
[42] SNOOK, K. 和 ARNEY, D. (2002) 选择的连续体：本科数学的教学技巧。在：核心数学跨学科研讨会论文集：考虑本科数学前两年的变化。华盛顿哥伦比亚特区：美国数学协会，第 143-149 页。
[43] TEZER, M. (2019) 数学建模在 STEM 整合和教育中的作用。在：FOMUNYAM, K.G. (编。) 将 21 世纪的干教育理论化。英泰开放。可从 https://www.intechopen.com/chapters/68547 获得。