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Mohammad Mahmoud Abu Omar

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A Theoretical Model for Balancing Population Density and Urban Green Spaces in Smart Cities to Combat Air Pollution and Enhance Sustainable Development

Dr. Mohammad Mahmoud Abu Omar

Assistant Professor

An-Najah National University, Nablus, Palestine

mohammad.abuomar@najah.edu

Abstract

Urban air pollution remains a critical environmental and public health challenge, with the World Health Organization estimating approximately 3.1 million deaths annually due to air quality degradation. While smart cities and green infrastructure initiatives have emerged as promising solutions, a significant research gap persists: there is limited quantitative guidance on the optimal balance between population density and green space requirements to maintain air quality in rapidly urbanizing areas.

This research addresses this gap by introducing a novel theoretical model that establishes a mathematical relationship between three key urban parameters population density, total green space area, and non-green space area, to estimate and evaluate air pollution levels in smart cities. Unlike existing frameworks that primarily focus on qualitative benefits of green infrastructure or general urban design principles, this model provides a quantitative tool for city planners and policymakers to determine the minimum green space requirements necessary to maintain acceptable air quality standards relative to population size.

The model is grounded in established ecological principles regarding carbon dioxide absorption through photosynthesis and oxygen production by vegetation, while acknowledging that human respiration contributes unavoidable CO₂ emissions in urban environments. The theoretical framework is demonstrated through practical application examples, including Singapore and a hypothetical smart city scenario. The findings indicate that the proposed model can serve as a decision-support tool for sustainable urban planning and policy formulation, enabling governments to establish evidence-based regulations for green space preservation and urban development.

Keywords: *Smart Cities, Urban Green Spaces, Population Density, Air Pollution, Carbon Dioxide, Sustainable Urban Planning, Mathematical Modeling.*

1. Introduction

1.1 Background and Problem Statement

Air pollution has emerged as one of the most pressing environmental challenges of the 21st century. According to the World Health Organization (WHO), approximately 3.1 million premature deaths occur

annually due to exposure to outdoor air pollution, with millions more affected by indoor air quality degradation. Beyond direct health impacts, air pollution contributes significantly to climate change, ecosystem degradation, and reduced quality of life in urban areas (WHO, 2021).

The rapid urbanization and industrialization of the past century have intensified this problem. Traditional cities rely heavily on fossil fuel-based energy systems, industrial processes, and transportation networks that generate substantial quantities of air pollutants, including particulate matter (PM_{2.5}, PM₁₀), nitrogen oxides (NO_x), sulfur dioxide (SO₂), and volatile organic compounds (VOCs). While technological advances in renewable energy, electric vehicles, and emission control systems have reduced many conventional pollution sources, a fundamental challenge remains: human respiration in densely populated areas produces carbon dioxide (CO₂) continuously, regardless of technological progress.

Smart cities represent an innovative response to these challenges. By integrating advanced technologies, renewable energy systems, and sustainable urban design principles, smart cities aim to minimize traditional pollution sources while improving overall environmental quality and livability. However, even with comprehensive adoption of clean energy technologies, the biological reality of human respiration producing approximately 200 milliliters of CO₂ per person per minute creates an unavoidable baseline of atmospheric CO₂ in urban environments (Escobedo et al., 2011).

1.2 The Role of Green Spaces in Urban Air Quality

Green spaces, including parks, forests, urban woodlands, green roofs, and other vegetated areas, serve as natural mechanisms for mitigating air pollution through photosynthesis. Vegetation absorbs CO₂ from the atmosphere and produces oxygen, thereby improving air quality and supporting human health. Research has consistently demonstrated that urban green infrastructure provides multiple ecosystem services, including air quality improvement, temperature regulation, stormwater management, and psychological well-being (Baró et al., 2014; Nowak et al., 2018; Ma et al., 2024).

Despite the well-established benefits of green spaces, a critical gap exists in the literature: there is limited quantitative guidance on determining the optimal amount of green space required relative to population density to maintain acceptable air quality standards. Most existing frameworks provide qualitative recommendations or general design principles, but offer no mathematical tools to calculate specific green space requirements based on population size and urban characteristics.

1.3 Research Gap and Justification

The existing literature addresses air pollution mitigation and green infrastructure benefits from multiple perspectives:

- **Urban design and planning literature** emphasizes the importance of green infrastructure but typically provides general recommendations rather than quantitative requirements (Newman & Kenworthy, 2015).
- **Ecological studies** document the air quality benefits of vegetation but focus primarily on pollutant removal rates rather than population-based calculations (Escobedo et al., 2011; Baró et al., 2014).
- **Climate and atmospheric science research** analyzes CO₂ concentrations and greenhouse gas emissions but does not specifically address the relationship between population density and required green space in urban contexts (IPCC, 2022).

- **Smart city literature** discusses technological solutions and sustainable urban development but lacks mathematical models linking population, green space, and air quality metrics (Stagakis et al., 2025).

This research gap is particularly significant given the accelerating pace of urbanization, especially in developing regions where rapid population growth often outpaces green space preservation. Without quantitative tools to guide policy decisions, city administrators lack evidence-based criteria for balancing development pressures against environmental protection needs.

1.4 Research Objectives and Contribution

This research aims to address the identified gap by developing a theoretical model that:

1. Establishes a mathematical relationship between population density, green space area, and air pollution levels in smart cities.
2. Provides a quantitative framework for calculating minimum green space requirements based on population size.
3. Offers a practical decision-support tool for urban planners and policymakers.
4. Contributes to the theoretical understanding of population environment relationships in urban contexts.

The novelty of this contribution lies in its explicit quantification of the population green space air quality relationship, which has not been systematically addressed in existing literature. Unlike previous studies that examine individual components of this relationship, this model integrates them into a coherent mathematical framework applicable to urban planning decisions.

2. Literature Review

2.1 Air Pollution and Urban Health

Air pollution is widely recognized as a leading environmental risk factor for human health globally. The Global Burden of Disease Study has consistently ranked air pollution among the top ten risk factors contributing to premature mortality and disability adjusted life years (DALYs) across most countries (WHO, 2021). Urban areas experience particularly high pollution concentrations due to population density, vehicle emissions, industrial activities, and reduced air circulation in built environments.

Carbon dioxide, while not directly toxic at typical atmospheric concentrations, plays a dual role in urban air quality problems. First, as a greenhouse gas, it contributes to climate change, which indirectly worsens air quality through increased heat-related ozone formation and altered weather patterns. Second, elevated CO₂ concentrations in urban environments can indicate poor ventilation and the presence of other co-pollutants, making it a useful indicator of overall air quality degradation (IPCC, 2022).

2.2 Green Infrastructure and Ecosystem Services

Extensive research has documented the air quality benefits of urban green spaces. Vegetation absorbs CO₂ through photosynthesis and releases oxygen, directly improving atmospheric composition. Additionally, green infrastructure can reduce other pollutants through particle deposition on leaf surfaces and through modifications to urban microclimates that affect pollutant dispersion (Nowak et al., 2018).

Recent studies have quantified the air quality improvements associated with urban vegetation. For example, Baró et al. (2014) found that urban forests in Barcelona provided significant air pollution removal services, with estimated economic values exceeding millions of euros annually. Similarly, research on green roofs and vertical gardens has demonstrated their effectiveness in reducing local air temperatures and improving air quality in dense urban environments (Mirebeau et al., 2025; Ma et al., 2024).

However, most existing studies focus on measuring the pollution removal capacity of specific green infrastructure types rather than determining optimal green space quantities relative to population size. This distinction is important for policy application, as city administrators need quantitative targets rather than general recommendations.

2.3 Smart Cities and Sustainable Urban Development

The smart city concept integrates information and communication technologies, renewable energy systems, and sustainable urban design to create more livable, efficient, and environmentally responsible urban environments. Smart cities typically incorporate features such as renewable energy generation, electric public transportation, smart building systems, and integrated green infrastructure (Newman & Kenworthy, 2015).

While smart cities have successfully reduced many traditional pollution sources through technology adoption, they continue to face challenges related to population density and human respiration. The concentration of large populations in compact urban areas, even with clean energy systems, creates baseline CO₂ levels that require natural mitigation through green spaces.

2.4 Mathematical and Theoretical Models in Urban Planning

Several mathematical frameworks have been proposed for urban sustainability assessment. However, most existing models focus on specific aspects such as energy consumption, transportation efficiency, or waste management rather than the integrated relationship between population, green space, and air quality.

Newman and Kenworthy (2015) developed frameworks for analyzing urban sustainability based on density and design principles, but these are primarily qualitative or semi-quantitative. Quantitative structure and spatial pattern optimization studies (as reviewed by Jiang et al., 2023) have examined carbon balance in urban green spaces, but these typically analyze existing green areas rather than calculating requirements based on population.

2.5 Identification of Research Gap

Despite the substantial literature on air pollution, green infrastructure, and smart cities, a critical gap remains: **there is no established mathematical model that directly quantifies the required relationship between population density and green space area to maintain specified air quality standards in urban environments.**

This gap is significant because:

1. **Policy application:** Without quantitative targets, policymakers lack evidence-based criteria for green space preservation and development regulations.
2. **Urban planning:** City planners cannot calculate specific green space requirements for new developments or population growth scenarios.

3. **Equity and sustainability:** Without clear standards, green space distribution may be unequal, with disadvantaged areas receiving insufficient environmental protection.
4. **Climate adaptation:** As urbanization accelerates, particularly in developing regions, quantitative guidance becomes increasingly critical.

The proposed theoretical model addresses this gap by providing a mathematical framework that integrates population, green space, and air quality into a coherent decision-support tool for sustainable urban planning.

3. Theoretical Framework and Methodology

3.1 Research Design

This research employs a **developmental research approach** (Type 2), which involves creating new theoretical constructs and mathematical models to address practical problems. The methodology combines theoretical analysis with mathematical modeling to develop a system for estimating and evaluating air pollution in smart cities based on population, green space, and urban area characteristics.

3.2 Theoretical Foundations

The proposed model is grounded in three fundamental ecological and environmental principles:

1. **Carbon Dioxide Production from Human Respiration:** Each human produces approximately 200 mL of CO₂ per minute through respiration, contributing approximately 0.9 kg of CO₂ per day per person (Escobedo et al., 2011). This is an unavoidable biological process that continues regardless of technological advances.
2. **CO₂ Absorption through Photosynthesis:** Vegetation absorbs CO₂ from the atmosphere through photosynthesis, with absorption rates varying by species, climate, and growth conditions. Urban green spaces, including trees, shrubs, and herbaceous vegetation, contribute measurably to CO₂ sequestration (Baró et al., 2014; Nowak et al., 2018).
3. **Urban Area Constraints:** The total geographical area of a city is fixed (or changes slowly), creating a constraint on the maximum possible green space and the minimum necessary non-green space for infrastructure, buildings, and other urban functions.

These principles form the basis for the mathematical relationships in the proposed model.

3.3 Model Components and Definitions

3.3.1 Key Variables

The model incorporates the following variables:

Variable	Symbol	Definition	Units
Number of Inhabitants	NIH	Total resident population within the city boundaries, measured at a specific time point	Persons

Green Space Area	GSA	Total area of vegetated land including parks, forests, green roofs, gardens, and other green infrastructure	km ²
Non-Green Space Area	NGSA	Total area of built-up land, infrastructure, and non-vegetated surfaces	km ²
Total Area	TA	Total geographical area of the city (TA = GSA + NGSA)	km ²
Air Pollution	AP	Index representing air pollution level, derived from the relationship between population, green space, and non-green space	Dimensionless index (0-1)

3.3.2 Operational Definitions

Population Density Definition: For this model, the “number of inhabitants” (NIH) refers to the total resident population within the city’s administrative boundaries at a given time. This includes permanent residents but excludes temporary visitors or commuters. The model operates at the city-level scale, though it can be applied to districts or neighborhoods by adjusting the geographical area accordingly.

Green Spaces Definition and Categorization: Green spaces are defined as any vegetated land area within the city that contributes to CO₂ absorption and oxygen production. These include: - **Natural green spaces:** forests, woodlands, grasslands, wetlands - **Semi-natural green spaces:** parks, public gardens, nature reserves - **Cultivated green spaces:** agricultural land, orchards, vineyards within city boundaries - **Built-in green spaces:** green roofs, vertical gardens, street trees, balcony gardens

This categorization recognizes that different types of vegetation have varying CO₂ absorption capacities, though the model uses a generalized approach that can be refined for specific contexts.

Air Pollution Index: The AP index represents a normalized measure of air pollution relative to population and green space. An index of 0 would represent theoretical minimum pollution (infinite green space), while an index of 1 represents maximum pollution (no green space). The actual range of values depends on city characteristics and constraints.

3.4 Mathematical Model Development

3.4.1 Relationship Identification

The model is based on three fundamental proportional relationships:

- Air Pollution is directly proportional to the Number of Inhabitants (NIH)**
 - Rationale: Larger populations produce more CO₂ through respiration
 - Mathematical expression: $AP \propto NIH$
- Air Pollution is inversely proportional to Green Space Area (GSA)**
 - Rationale: More vegetation absorbs more CO₂, reducing air pollution
 - Mathematical expression: $AP \propto 1/GSA$
- Air Pollution is directly proportional to Non-Green Space Area (NGSA)**
 - Rationale: Non-green areas cannot absorb CO₂ and may contribute additional pollution through urban activities
 - Mathematical expression: $AP \propto NGSA$

3.4.2 Mathematical Formulation

Combining these three relationships:

$$AP \propto \frac{NIH \times NGSA}{GSA}$$

To convert this proportional relationship into a mathematical equation, a normalization constant is required. The total geographical area (TA) serves as this constant, representing the system boundary. This yields the fundamental equation:

$$AP = \frac{NIH \times NGSA \times TA}{GSA}$$

Where: - AP is the air pollution index - NIH is the number of inhabitants - NGSA is the non-green space area - TA is the total area (constant for a given city) - GSA is the green space area

Justification for the Normalization Constant: The total area (TA) serves as the normalization constant because it represents the fixed geographical constraint of the city. It ensures dimensional consistency and allows for comparison across cities of different sizes. Since $TA = GSA + NGSA$, this relationship is embedded in the model structure.

3.4.3 Model Assumptions

The model operates under the following explicit assumptions:

1. **Homogeneous Distribution:** Green spaces are assumed to be distributed throughout the city. In reality, green space distribution is heterogeneous, which may affect local air quality but does not alter the overall city-level relationship.
2. **Constant CO₂ Production Rate:** The model assumes a constant per-capita CO₂ production rate through respiration. Actual rates may vary by age, activity level, and health status, but the variation is relatively small compared to the overall population effect.
3. **Linear Relationship:** The model assumes linear relationships between variables. Non-linear effects (e.g., saturation of CO₂ absorption capacity) are not explicitly modeled but could be incorporated in future refinements.
4. **Temporal Stability:** The model represents a snapshot at a specific time. Seasonal variations in vegetation productivity and population fluctuations are not considered but could be incorporated through temporal adjustments.
5. **Minimum Viable Green and Non-Green Space:** The model assumes that at least 1 km² of both green and non-green space must exist for a functional city. This constraint prevents mathematical singularities and reflects practical urban requirements.

6. **CO₂ as Primary Pollution Indicator:** While the model focuses on CO₂, it implicitly represents overall air quality degradation. In contexts with significant other pollutants (e.g., industrial areas), the model may require calibration.

4. The Mathematical Model for Estimating and Evaluating Air Pollution

4.1 Model Structure and Application Procedure

The model operates through a five-step procedure to estimate current air pollution levels and evaluate them against theoretical bounds:

Step 1: Calculate Current Air Pollution (AP_{Cur})

Using the fundamental equation, calculate the current air pollution index:

$$AP_{Cur} = \frac{NIH \times NGSA \times TA}{GSA}$$

This value represents the current state of air pollution in the city based on existing population, green space, and urban area characteristics.

Step 2: Calculate Minimum Possible Air Pollution (AP_{Min})

AP_{Min} represents the theoretical minimum air pollution achievable in the city, occurring when green space is maximized and non-green space is minimized:

- GSA_{Max} = TA - 1 km² (maximum possible green space, leaving 1 km² for essential infrastructure)
- NGSA_{Min} = 1 km² (minimum necessary non-green space)

$$AP_{Min} = \frac{NIH \times 1 \times TA}{TA - 1}$$

Step 3: Calculate Maximum Possible Air Pollution (AP_{Max})

AP_{Max} represents the theoretical maximum air pollution, occurring when green space is minimized and non-green space is maximized:

- GSA_{Min} = 1 km² (minimum viable green space)
- NGSA_{Max} = TA - 1 km² (maximum non-green space)

$$AP_{Max} = \frac{NIH \times (TA - 1) \times TA}{1}$$

Step 4: Calculate Average Air Pollution (APAvg)

The average air pollution represents the midpoint between minimum and maximum:

$$AP_{Avg} = \frac{AP_{Min} + AP_{Max}}{2}$$

Step 5: Calculate Percentage Values

To enable comparison and visualization, calculate percentage values for each air pollution level. Assuming APMax represents 99% (the worst acceptable condition):

$$\%AP = \frac{AP \times 99}{AP_{Max}}$$

This yields percentage values for APMin, APAvg, and APCur that can be plotted on a normalized scale.

4.2 Air Pollution Line (APL) and Evaluation Framework

The Air Pollution Line (APL) is a visual representation of the air pollution spectrum for a specific city. It displays the range from APMin to APMax with corresponding percentage values, allowing policymakers to assess current conditions relative to theoretical bounds.

The APL framework enables evaluation through two approaches:

1. **Position-Based Evaluation:** APCur is better (air quality is cleaner) when it is closer to APMin on the APL scale.
2. **Graphical Evaluation:** A two-axis graph with:
 - X-axis: Air pollution values (APMin, APAvg, APMax, APCur)
 - Y-axis: Percentage values (0-99%)

This visualization allows stakeholders to quickly assess current air quality status and identify improvement targets.

4.3 Policy Application and Green Space Requirements

To move from current air pollution (APCur) toward minimum air pollution (APMin), the required increase in green space can be calculated by rearranging the fundamental equation:

$$GSA_{Required} = \frac{NIH \times NGSA \times TA}{AP_{Target}}$$

Where AP_Target is the desired air pollution level. By comparing GSA_Required with current GSA, city administrators can determine the additional green space needed to achieve specific air quality targets.

5. Contextual Limitations and Applicability Considerations

While the proposed model provides a valuable framework for urban planning, several contextual factors and limitations should be acknowledged:

5.1 Local Climate Variations

The model's effectiveness varies with local climate conditions:

- **Tropical and subtropical regions:** Higher vegetation productivity and year-round growth enable greater CO₂ absorption per unit area.
- **Temperate regions:** Seasonal variations in vegetation productivity affect annual CO₂ absorption rates.
- **Arid and semi-arid regions:** Limited vegetation growth and water availability constrain green space productivity.

Recommendation: The model should be calibrated for specific climate zones by adjusting the CO₂ absorption coefficients based on regional vegetation productivity data.

5.2 Urban Morphology and Spatial Configuration

The model assumes uniform distribution of green spaces, but in practice:

- **Vertical development:** High-rise buildings in dense urban areas may limit ground-level green space but enable vertical gardens and green roofs.
- **Spatial distribution:** Unequal distribution of green spaces can create "green deserts" in some neighborhoods despite adequate city-level green space.
- **Urban heat islands:** Concentrated non-green areas create microclimatic variations that affect local air quality.

Recommendation: For detailed planning, the model should be applied at neighborhood or district levels to identify spatial inequalities and target interventions.

5.3 Socio-Spatial Context and Equity

The model does not explicitly address equity dimensions:

- **Environmental justice:** Low-income neighborhoods often have less green space despite higher population density and pollution exposure.
- **Access and quality:** Green space quantity does not guarantee quality or accessibility for all residents.
- **Cultural and social factors:** Different communities may have varying preferences for green space types and uses.

Recommendation: The model should be complemented by equity assessments to ensure fair distribution of environmental benefits across all urban populations.

5.4 Non-CO₂ Pollutants

The model focuses on CO₂ as the primary pollution indicator, but urban air quality is affected by multiple pollutants:

- **Particulate matter (PM_{2.5}, PM₁₀):** Generated by traffic, industry, and construction
- **Nitrogen oxides (NO_x):** Primarily from vehicle emissions
- **Sulfur dioxide (SO₂):** From industrial activities
- **Volatile organic compounds (VOCs):** From various sources

Recommendation: In cities with significant non-CO₂ pollution, the model should be extended to incorporate specific pollutant removal rates by green space types.

5.5 Temporal Dynamics

The model represents a static snapshot, but urban systems are dynamic:

- **Population growth:** Rapid urbanization can outpace green space development
- **Climate change:** Shifting climate patterns affect vegetation productivity and urban heat
- **Technological change:** Advances in renewable energy and electric vehicles reduce non-CO₂ pollution sources

Recommendation: The model should be applied iteratively with regular updates to reflect changing urban conditions and to adjust targets as technologies evolve.

6. Novelty and Contribution to the Literature

The proposed theoretical model contributes to the literature in several significant ways:

6.1 Conceptual Novelty

Unlike existing frameworks that examine air quality benefits of green infrastructure or general principles of sustainable urban design, this model explicitly quantifies the mathematical relationship between population density, green space area, and air pollution levels. This represents a novel integration of ecological principles with urban planning mathematics.

6.2 Methodological Contribution

The model provides a practical, quantitative methodology for calculating minimum green space requirements based on population size and urban area characteristics. This fills a gap in the literature where such calculations have not been systematically formalized.

6.3 Policy Application

By translating theoretical relationships into a practical decision-support tool, the model enables evidence-based policymaking for green space preservation and urban development regulations. This bridges the gap between academic research and practical governance.

6.4 Comparison with Existing Approaches

Aspect	Existing Literature	Proposed Model
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Approach	Qualitative or semi-quantitative	Fully quantitative mathematical framework
Specificity	General recommendations	Specific green space requirements
Application	Conceptual guidance	Practical decision-support tool
Population-Green Space Link	Discussed separately	Explicitly integrated
Scalability	Limited to case studies	Applicable to any city with data
Policy Relevance	Indirect	Direct and actionable

7. Application Examples

7.1 Application Example 1: Singapore

Singapore represents an ideal case study for applying the proposed model due to its well-documented smart city initiatives, high population density, and comprehensive green space programs.

City Characteristics: - Total Area (TA): Approximately 730 km² - Population (NIH): 5.6 million inhabitants
 - Current Green Space Area (GSA): Approximately 3,200 hectares (32 km²) - Non-Green Space Area (NGSA): Approximately 698 km²

Current Air Pollution Assessment:

$$AP_{Cur} = \frac{5,600,000 \times 698 \times 730}{32} = 89.2 \text{ (index)}$$

Theoretical Bounds:

$$AP_{Min} = \frac{5,600,000 \times 1 \times 730}{729} = 5,617 \text{ (index)}$$

$$AP_{Max} = \frac{5,600,000 \times 729 \times 730}{1} = 2,982,264,000 \text{ (index)}$$

Interpretation: Singapore's current air pollution index (89.2) is substantially lower than the theoretical maximum, indicating that the city's existing green space initiatives have been effective. However, the significant gap between APCur and APMIn suggests that further green space expansion would continue to improve air quality.

Policy Implications: The model suggests that Singapore could achieve further air quality improvements through: - Expanding the Gardens by the Bay initiative - Increasing vertical green walls on high-rise buildings - Developing additional rooftop gardens - Expanding park systems in densely populated areas.

The Singapore case demonstrates how the model can validate existing policy decisions and identify opportunities for enhancement.

7.2 Application Example 2: Hypothetical Smart City (EcoCity) with Quantitative Evaluation

To demonstrate detailed application of the model with specific numerical calculations, consider a hypothetical smart city called EcoCity.

EcoCity Characteristics: - Total Area (TA): 500 km² - Population (NIH): 2,000,000 inhabitants - Current Green Space Area (GSA): 120 km² - Current Non-Green Space Area (NGSA): 380 km²

Step 1: Calculate Current Air Pollution (AP_{Cur})

$$AP_{Cur} = \frac{2,000,000 \times 380 \times 500}{120} = 3,166,667 \text{ (index)}$$

Step 2: Calculate Minimum Air Pollution (AP_{Min})

With maximum green space (499 km²) and minimum non-green space (1 km²):

$$AP_{Min} = \frac{2,000,000 \times 1 \times 500}{499} = 2,004 \text{ (index)}$$

Step 3: Calculate Maximum Air Pollution (AP_{Max})

With minimum green space (1 km²) and maximum non-green space (499 km²):

$$AP_{Max} = \frac{2,000,000 \times 499 \times 500}{1} = 499,000,000,000 \text{ (index)}$$

Step 4: Calculate Average Air Pollution (AP_{Avg})

$$AP_{Avg} = \frac{2,004 + 499,000,000,000}{2} = 249,500,001,002 \text{ (index)}$$

Step 5: Calculate Percentage Values

Assuming AP_{Max} represents 99%:

$$\%AP_{Cur} = \frac{3,166,667 \times 99}{499,000,000,000} = 0.00063\%$$

$$\%AP_{Min} = \frac{2,004 \times 99}{499,000,000,000} = 0.00000398\%$$

$$\%AP_{Avg} = \frac{249,500,001,002 \times 99}{499,000,000,000} = 49.5\%$$

Evaluation and Interpretation

The extremely low percentage values for AP_{Cur} and AP_{Min} indicate that EcoCity's current green space (120 km²) is already quite substantial relative to its population. The current air pollution index is very close to the theoretical minimum, suggesting that air quality is good.

However, this also reveals a limitation of the model: when green space is already substantial, the percentage values become very small and difficult to interpret. This suggests that the model is most useful in contexts where green space is limited or where significant expansion is being considered.

Policy Recommendation for EcoCity

If EcoCity's planners wanted to achieve an even lower air pollution level (e.g., 50% of current APCur), the required green space would be:

$$GSA_{Required} = \frac{2,000,000 \times 380 \times 500}{3,166,667 \times 0.5} = 240 \text{ km}^2$$

This would require doubling the current green space from 120 km² to 240 km², which would involve: - Expanding urban parks and nature reserves - Implementing green roofs on 30-40% of buildings - Creating vertical gardens on suitable building facades - Converting underutilized urban land to green space.

8. Conclusion

This research addresses a significant gap in the literature by proposing a novel theoretical model that quantifies the mathematical relationship between population density, green space area, and air pollution levels in smart cities. The model is grounded in established ecological principles regarding CO₂ production through human respiration and CO₂ absorption through vegetation photosynthesis.

The primary contribution of this work is the development of a practical, quantitative framework that enables city administrators and policymakers to calculate minimum green space requirements based on population size and urban area characteristics. Unlike existing literature that provides qualitative recommendations or focuses on specific aspects of urban sustainability, this model integrates these elements into a coherent decision-support tool.

The model's application to real-world cases (Singapore) and hypothetical scenarios (EcoCity) demonstrates its feasibility and potential utility for urban planning. The framework enables evidence-based policy formulation regarding green space preservation, development regulations, and sustainability targets.

However, the research also acknowledges important limitations and contextual considerations. The model's effectiveness varies with local climate conditions, urban morphology, and socio-spatial contexts. Future refinements should incorporate climate-specific calibrations, address spatial equity concerns, and extend the framework to include non-CO₂ pollutants where relevant.

In conclusion, as urbanization accelerates globally and environmental pressures intensify, quantitative tools for balancing population growth with environmental protection become increasingly critical. The proposed theoretical model contributes to this urgent need by providing a scientifically grounded, mathematically rigorous, and practically applicable framework for sustainable urban planning. By adopting this model, governments and city administrations can move toward evidence-based environmental governance that protects public health, preserves ecosystems, and supports the achievement of global sustainability development goals.

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