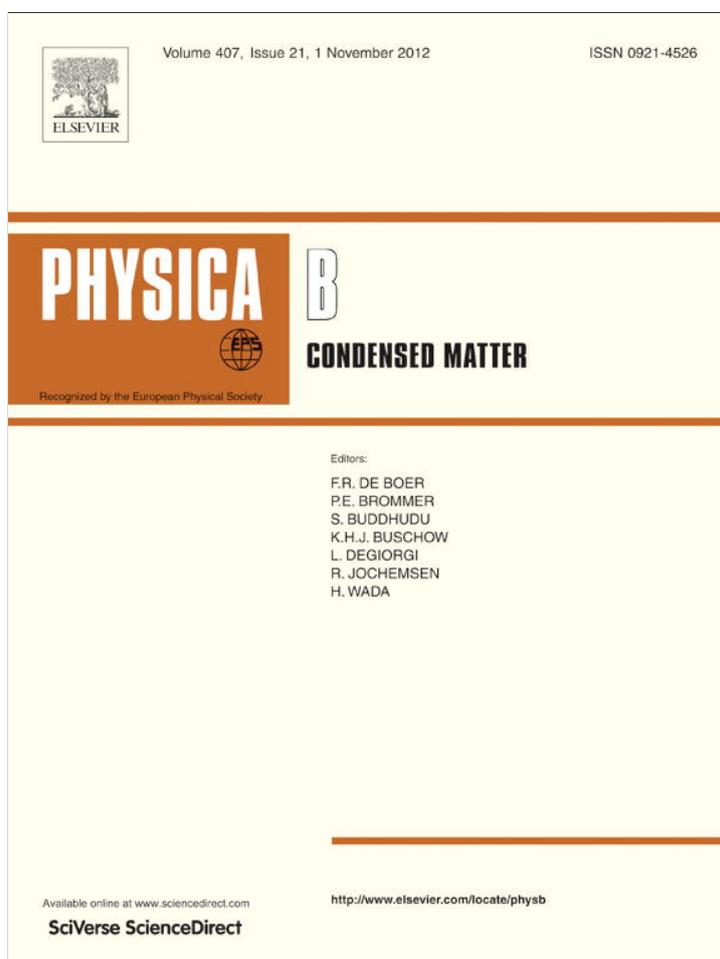


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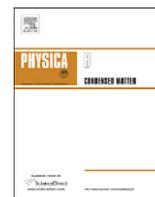
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# A quantum pseudodot system with two-dimensional pseudoharmonic oscillator in external magnetic and Aharonov-Bohm fields

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## ABSTRACT

Using the Nikiforov–Uvarov (NU) method, the energy levels and the wave functions of an electron confined in a two-dimensional (2D) pseudoharmonic quantum dot are calculated under the influence of temperature and an external magnetic field inside dot and Aharonov–Bohm (AB) field inside a pseudodot. The exact solutions for energy eigenvalues and wave functions are computed as functions of the chemical potential parameters, applied magnetic field strength, AB flux field, magnetic quantum number and temperature. Analytical expression for the light interband absorption coefficient and absorption threshold frequency are found as functions of applied magnetic field and geometrical size of quantum pseudodot. The temperature dependence energy levels for GaAs semiconductor are also calculated.

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## 1. Introduction

Over a long time, a considerable interest has been paid in studying size effects in orbital magnetism [1,2] and the magnetic properties of low-dimensional (2D) metallic and semiconducting structures with restricted geometries [3] on nanostructures such as dots, wires, wells, antidots, well wires and antiwells [4–6]. These structures can confine charge carriers in one, two and three dimensions. Experimental research is currently made to study the optical and quantum properties of low-dimensional semiconducting structures for the fabrication purposes and subsequent working of electronic and optical devices. More studies analyzing these structures have been focused on the interband light absorption coefficient in the spherical [7–9], parabolic, cylindrical and rectangular [10] quantum dots under the influence of magnetic field [11,12]. More other works on optical properties in nanostructures [13,14], band structure calculations, transport properties of Aharonov–Bohm (AB) type oscillations [15,16] and Althshuler–Aharonov–Spivak type oscillation [17,18].

The quantum antidot structure has been modeled using the repulsive antidot harmonic oscillator with external magnetic and AB fields in cylindrical coordinates to obtain an exact bound state

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solutions for the Schrödinger equation. In addition, the influence of dots and antidots on thermodynamic properties (e.g., magnetization) of the system, the magnetic transport properties and also the magneto-optical (MO) spectroscopic characteristics of a 2D electron gas in a magnetic field are studied [19]. The nature of MO transitions in this system demonstrate the appearance of rich spectrum of nonequidistant frequencies are different from the MO spectrum for a dot modeled by a harmonic potential. The quantum antidot is modeled as an electron moving outside a cylinder of radius  $a$  in the presence of magnetic and AB flux fields to find analytical expressions for energy and wave function [20]. The intensive investigations have shown that optoelectronic properties of quantum dots are quite sensitive to the reduction of their dimensionality and to the strength of applied external magnetic field, and depend strongly on the electron–electron interaction. The numerical and analytical solutions obtained for the dynamics of two classical electrons interacting via a Coulomb field in a 2D antidot superlattice potential in the presence of crossed electric and magnetic fields are quite different than the noninteracting electrons [21]. Some authors have studied a 2D theoretical model for the quantum dots in which electrons were confined by a nonhomogenous magnetic field (the so-called magnetic antidot) [22]. The pseudoharmonic (PH) interaction [23,24] is used in modeling the quantum dots (QDs) and quantum antidots (QADs) in the presence of a strong magnetic field together with an AB flux field in nanostructures [25]. The spectral properties in a 2D electron confined by a pseudoharmonic

quantum dots (PHQDs) potential under the influence of a uniform magnetic field  $\vec{B}$  along the  $z$  direction and AB flux field created by a solenoid inserted inside the pseudodot have been studied [25]. The electron–phonon interaction on the surface of a sphere has been investigated in the presence of a uniform strong magnetic field [9]. The energies of ground and excited states have been analyzed with respect to electron–phonon coupling constant, sphere radius and dimensionless magnetic field [9]. Furthermore, the phonon interactions to the energy levels of a 2D electron confined by a parabolic potential in the presence of an antidot potential that produces a radially symmetrical hole have been studied in the presence of a uniform magnetic field along the  $z$  direction and AB flux created by a solenoid inserted inside the antidot [26].

It is well-known that factors such as impurity, electric and magnetic fields, pressure, and temperature play important roles in the electronic, optical and transport properties of low-dimensional semiconductor nanostructures [4,27–32]. Hence, many works in 2D quantum dots and semiconductors are studied under the influence of external magnetic field [33–38]. For example, the set of energy eigenstates of a 2D anisotropic harmonic potential in a uniform magnetic field is found [33]. The formation of dark states and the AB effect have been studied in symmetrically/asymmetrically coupled three- and four-quantum dot systems. Without a transverse magnetic field, destructive interference can trap an electron in a dark state. However, the introduction of a transverse magnetic field can disrupt the dark state giving rise to oscillation in current [34]. The propagator for an electron moving in 2D quadratic saddle-point potential has been studied in the presence of a perpendicular uniform magnetic field [35]. The electron states in the 2D GaAs/AlGaAs quantum ring are theoretically studied in effective mass approximation taking into account on-center donor impurity and uniform magnetic field perpendicular to the ring plane [36]. Magneto transport properties of 2D electron gas in AlGaIn/AlN/GaN heterostructures have been studied [37]. The binding energy of a hydrogenic impurity in self-assembled double quantum dots is calculated via the finite-difference method. The variation in binding energy with donor position, structure parameters and external magnetic field is studied [38].

Over the past years, the NU method [39] has shown to be a powerful tool in solving the second-order differential equation. It was applied successfully to a large number of potential models [40–43]. This method has also been used to solve the Schrödinger equation [40], relativistic spin-0 KG equation [41], relativistic spin-1/2 Dirac equation [42] and spinless Salpeter equation [43] with different potential models. Recently, an alternative treatment is proposed for the calculations carried out within the frame of the NU method which removes a drawback in the original theory and bypasses some difficulties in solving the Schrödinger equation [44]. This formalism has also been extended to the relativistic wave equations [45]. The low-lying energy levels of two interacting electrons confined in a 2D parabolic quantum dot have been studied in magnetic field [45].

Therefore, we carry out detailed exact energetic spectrum and wave functions of the Schrödinger equation with a pseudoharmonic potential in the presence of external magnetic and AB flux fields through the NU method [39]. The low-lying energy levels serve as a base for calculating the corresponding interband light (optical) absorption coefficient and the threshold frequency value of absorption for the given model. In addition, the effect of the temperature on the effective mass is also calculated for GaAs semiconductor.

The paper is organized as follows. In Section 2, we briefly present the basic formulas of the NU method. In Section 3, the quantum dots and antidots with the pseudoharmonic interaction

are studied under the influence of external magnetic and AB flux fields. The exact analytical expressions for the energy spectrum and wave function are calculated. The interband light absorption coefficient and temperature dependence of effective mass are also investigated. Results and discussions are performed in Section 4. Finally, we give our concluding remarks in Section 5.

## 2. The Nikiforov–Uvarov method

The NU is usually used in solving a second-order hypergeometric-type differential equations satisfying special orthogonal functions [39–43]. In spherical or cylindrical coordinates, the resulting Schrödinger-like equation with a given potential is reduced to a hypergeometric type equation through making a suitable change of variables, say,  $r \rightarrow z$  and then solved systematically for its exact or approximate eigensolutions (energy levels and wave functions). The second-order hypergeometric equation takes the form [39]

$$\sigma^2(z)g_{nl}''(z) + \sigma(z)\tilde{\tau}(z)g_{nl}'(z) + \tilde{\sigma}(z)g_{nl}(z) = 0, \quad (1)$$

where  $\sigma(z)$  and  $\tilde{\sigma}(z)$  are at most second-degree polynomials and  $\tilde{\tau}(z)$  is a first-degree polynomial. The primes denote derivatives with respect to  $z$ . To find a particular solution of Eq. (1), one can use the wave functions,  $g_{nl}(z)$ , as

$$g_{nl}(z) = \phi(z)y_{nl}(z), \quad (2)$$

to recast Eq. (1) into the hypergeometric-type equation

$$\sigma(z)y_{nl}''(z) + \tau(z)y_{nl}'(z) + \lambda y_{nl}(z) = 0, \quad (3)$$

where

$$\lambda = k + \pi'(z), \quad (4)$$

and  $y_{nl}(z)$  satisfies the Rodrigues relation:

$$y_{nl}(z) = \frac{A_n}{\rho(z)} \frac{d^n}{dz^n} [\sigma^n(z)\rho(z)]. \quad (5)$$

In the above equation,  $A_n$  is a constant related to the normalization and  $\rho(z)$  is the weight function satisfying the condition

$$[\sigma(z)\rho(z)]' = \tau(z)\rho(z), \quad (6)$$

with

$$\tau(z) = \tilde{\tau}(z) + 2\pi(z), \quad \tau'(z) < 0. \quad (7)$$

Since  $\rho(z) > 0$  and  $\sigma(z) > 0$ , the derivative of  $\tau(z)$  has to be negative [39] which is the main essential condition in the choice of particular solution relevant to the real bound state solution. The other part of wave functions in Eq. (2) is the solution of the logarithmic equation:

$$\frac{\phi'(z)}{\phi(z)} = \frac{\pi(z)}{\sigma(z)}, \quad (8)$$

where

$$\pi(z) = \frac{1}{2}[\sigma'(z) - \tilde{\tau}(z)] \pm \sqrt{\frac{1}{4}[\sigma'(z) - \tilde{\tau}(z)]^2 - \tilde{\sigma}(z) + k\sigma(z)} \quad (9)$$

is a polynomial of order one. The determination of  $k$  is the essential point in the calculation of  $\pi(z)$ , for which the discriminant of the square root in the last equation is set to zero. This gives the polynomial  $\pi(z)$  which is dependent on the transformation function  $z(r)$ . Also, the parameter  $\lambda$  defined in Eq. (4) takes the form

$$\lambda = \lambda_n = -n\tau'(z) - \frac{1}{2}n(n-1)\sigma''(z), \quad n = 0, 1, 2, \dots \quad (10)$$

To obtain the energy formula, we need to establish a relationship between  $\lambda$  and  $\lambda_n$ , by means of Eqs. (4) and (10).

### 3. QDs and QADs in external fields

#### 3.1. Exactly solvable bound states

Consider a 2D single charged electron,  $e$ , with an effective mass,  $\mu$ , interacting via a radially symmetrical dot (electron) and antidot (hole) potential in a uniform magnetic field,  $\vec{B} = B\hat{z}$  and an AB flux field, applied simultaneously. The Schrödinger equation with interaction potential field has the form [46]

$$\left[ \frac{1}{2\mu} \left( \vec{p} + \frac{e}{c} \vec{A} \right)^2 + V_{\text{conf}}(r) \right] \psi(r, \phi) = E\psi(r, \phi), \quad (11)$$

where  $E$  is the energy eigenvalues,  $\vec{p} = -i\hbar\vec{\nabla}$  is the momentum,  $\mu$  is the effective mass of an electron and  $V_{\text{conf}}(r)$  is the scalar pseudoharmonic interaction defined by [23,24]

$$V_{\text{conf}}(r) = V_0 \left( \frac{r}{r_0} - \frac{r_0}{r} \right)^2, \quad (12)$$

with  $r_0$  and  $V_0$  are the zero point (effective radius) and the chemical potential. Besides, the vector potential  $\vec{A}$  in Eq. (11) may be represented as a sum of two terms,  $\vec{A} = \vec{A}_1 + \vec{A}_2$  having the azimuthal components [25]

$$\vec{A}_1 = \frac{Br}{2} \hat{\phi}, \quad \vec{A}_2 = \frac{\Phi_{AB}}{2\pi r} \hat{\phi}, \quad \vec{A} = \left( \frac{Br}{2} + \frac{\Phi_{AB}}{2\pi r} \right) \hat{\phi}. \quad (13a)$$

$$\vec{\nabla} \times \vec{A}_1 = \vec{B}, \quad \vec{\nabla} \times \vec{A}_2 = 0, \quad (13b)$$

where  $\vec{B} = B\hat{z}$  is the applied magnetic field and  $\vec{A}_2$  describes the additional magnetic flux  $\Phi_{AB}$  created by a solenoid inserted inside the antidot (pseudodot). Let us take the wave function  $\psi(\vec{r}, \phi)$  in cylindrical coordinates as

$$\psi(r, \phi) = \frac{1}{\sqrt{2\pi}} e^{im\phi} g(r), \quad m = 0, \pm 1, \pm 2, \dots, \quad (14)$$

where  $m$  is the magnetic quantum number. Inserting the wave functions (14) into the Schrödinger equation (11), we obtain a second-order differential equation satisfying  $g(r)$ ,

$$g''(r) + \frac{1}{r} g'(r) + \frac{1}{r^2} (-\gamma^2 r^4 + v^2 r^2 - \beta^2) g(r) = 0, \quad (15)$$

with

$$v^2 = \frac{2\mu}{\hbar^2} (E + 2V_0) - \frac{\mu\omega_c}{\hbar} (m + \xi), \quad (16a)$$

$$\beta^2 = (m + \xi)^2 + a^2, \quad (16b)$$

$$\gamma^2 = \frac{2\mu V_0}{\hbar^2 r_0^2} + \left( \frac{\mu\omega_c}{2\hbar} \right)^2, \quad (16c)$$

where  $\xi = \Phi_{AB}/\Phi_0$  is taken as integer with the flux quantum  $\Phi_0 = hc/e$ ,  $\omega_c = eB/\mu c$  is the cyclotron frequency and  $a = k_F r_0$  with  $k_F = \sqrt{2\mu V_0/\hbar^2}$  is the fermi wave vector of the electron. The magnetic quantum number  $m$  relates to the quantum number  $|\beta|$  [Eq. (16b)].<sup>1</sup> Further, the radial wave function  $g(r)$  has to satisfy the asymptotic behaviours, that is,  $g(0) \rightarrow 0$  and  $g(\infty) \rightarrow 0$ . To make the solution of Eq. (15) amendable by NU method, it is necessary to introduce the following change of variables  $s = r^2$ , mapping  $r \in (0, \infty)$  into  $s \in (0, \infty)$  which in turn recasts Eq. (15) into the hypergeometric form (1) as

$$g''(s) + \frac{2}{(2s)} g'(s) + \frac{1}{(2s)^2} (-\gamma^2 s^2 + v^2 s - \beta^2) g(s) = 0. \quad (17)$$

<sup>1</sup> For this system, only two independent integer quantum numbers are required.

Applying the basic ideas of Ref. [39], by comparing Eq. (17) with Eq. (1) gives us the essential polynomials:

$$\tilde{\tau}(s) = 2, \quad \sigma(s) = 2s, \quad \tilde{\sigma}(s) = -\gamma^2 s^2 + v^2 s - \beta^2, \quad (18)$$

and substituting the polynomials given by Eq. (18) into Eq. (9), we obtain  $\pi(s)$  as

$$\pi(s) = \pm \frac{1}{2} \sqrt{\gamma^2 s^2 + (2k - v^2)s + \beta^2}. \quad (19)$$

The expression under the square root of the above equation must be the square of a polynomial of first degree. This is possible only if its discriminant is zero and the constant parameter (root)  $k$  can be found by the condition that the expression under the square root has a double zero. Hence,  $k$  is being obtained as  $k_{+,-} = v^2/2 \pm \beta\gamma$ . In that case, it can be written in the four possible forms of  $\pi(s)$ ;

$$\pi(s) = \begin{cases} +(\gamma s \pm \beta) & \text{for } k_+ = \frac{1}{2}v^2 + \beta\gamma, \\ -(\gamma s \pm \beta) & \text{for } k_- = \frac{1}{2}v^2 - \beta\gamma. \end{cases} \quad (20)$$

One of the four possible forms of  $\pi(s)$  must be chosen to obtain an energy spectrum formula. Therefore, the most suitable physical choice is

$$\pi(s) = \beta - \gamma s,$$

for  $k_-$ . The trick in this choice provides the negative derivative of  $\tau(s)$  as required in Eq. (7). Hence,  $\tau(s)$  and  $\tau'(s)$  are obtained as

$$\tau(s) = 2(1 + \beta) - 2\gamma s, \quad \tau'(s) = -2\gamma < 0. \quad (21)$$

In this case, a new eigenvalue equation becomes

$$\tilde{\lambda}_n = 2\gamma n, \quad n = 0, 1, 2, \dots, \quad (22)$$

where  $\tilde{\lambda}_n = -n\tau'(s) - (n(n-1)/2)\sigma''(s)$  has been used and  $n$  is the radial quantum number. Another eigenvalue equation is obtained from the equality  $\tilde{\lambda} = k + \pi'$  in Eq. (4),

$$\tilde{\lambda} = \frac{v^2}{2} - \gamma(\beta + 1). \quad (23)$$

In order to find an eigenvalue equation, the right-hand sides of Eqs. (22) and (23) must be compared with each other, i.e.,  $\tilde{\lambda}_n = \tilde{\lambda}$ . In this case the result obtained will depend on  $E_{nm}$  in the closed form:

$$v^2 = 2(2n + 1 + \beta)\gamma. \quad (24)$$

Upon the substitution of the terms of right-hand sides of Eqs. (16a)–(16c) into Eq. (24), we can immediately arrive at the energy spectrum formula in the presence of PH potential

$$E_{nm}(\xi, \beta) = \hbar\Omega \left( n + \frac{|\beta| + 1}{2} \right) + \frac{1}{2} \hbar\omega_c (m + \xi) - 2V_0, \quad \Omega = \sqrt{\omega_c^2 + 4\omega_D^2}, \quad (25)$$

where  $|\beta|$  is defined by Eq. (16b) and  $\omega_D = \sqrt{2V_0/\mu r_0^2}$ . We have two sets of quantum numbers  $(n, m, \beta)$  and  $(n', m', \beta')$  for dot (electron) and antidot (hole), respectively. Therefore, energetic spectrum formula (25) for the energy levels of the electron (hole) is identical to Eq. (7) of Ref. [25] and usually used to study the thermodynamics properties of quantum structures with dot and antidot in the presence and absence of magnetic field.

We consider a few special cases of our results:

- Ignoring the last  $-2V_0$  term, the above formula becomes the Bogachek–Landman [19] energy levels in the presence of dot and antidot potential.
- In the absence of pseudoharmonic quantum dot (PHQD), i.e.,  $V_0 = 0$ ,  $\Omega \rightarrow \omega_c$ , then  $E_{nm}(\xi) = \hbar\omega_c [n + \frac{1}{2}(|m + \xi| + 1)] + \frac{1}{2} \hbar\omega_c (m + \xi)$  which is the formula in the presence of  $B$  and  $\xi$  fields [19].

- When  $\xi = 0$  (i.e.,  $\Phi_{AB} = 0$ ), we find the Landau energy levels, i.e.,  $E_{nm} = \hbar\omega_c[n + \frac{1}{2}(|m| + m + 1)]$ .
- When both  $B=0$  ( $\omega_c = 0$ ) and  $\xi = 0$ , we find  $E_{nm} = (4\hbar V_0/\mu r_0^2)[n + (\sqrt{m^2 + 2\mu V_0 r_0^2/\hbar^2 + 1})/2] - 2V_0$ .
- When  $m=0$ , we have  $E_n = (4\hbar V_0/\mu r_0^2)(n + 1/2)$  for harmonic oscillator energy spectrum.

Let us calculate the corresponding wave functions. We find the first part of the wave function through Eq. (8), i.e.,

$$\phi_m(s) = \exp\left(\int \frac{\pi(s)}{\sigma(s)} ds\right) = s^{|\beta|/2} e^{-\gamma s/2}. \quad (26)$$

Then, the weight function defined by Eq. (6) as

$$\rho(s) = \frac{1}{\sigma(s)} \exp\left(\int \frac{\tau(s)}{\sigma(s)} ds\right) = s^{|\beta|} e^{-\gamma s}, \quad (27)$$

which gives the second part of the wave function (Rodrigues formula) via Eq. (5),

$$y_{n,m}(s) \sim s^{-|\beta|} e^{\gamma s} \frac{d^{nr}}{ds^{nr}} (s^{n+|\beta|} e^{-\gamma s}) \sim L_n^{(|\beta|)}(\gamma s), \quad (28)$$

where  $L_a^{(b)}(x) = ((a+b)!/a!b!)F(a,b+1;x)$  is the associated Laguerre polynomial and  $F(a,b;x)$  is the confluent hypergeometric function. The relation  $g(s) = \phi_m(s)y_{n,m}(s)$  gives the desired radial wave function as

$$g(r) = C_{n,m} r^{|\beta|} e^{-\gamma r^2/2} F(-n, |\beta| + 1; \gamma r^2), \quad (29)$$

and hence the total wave function from Eq. (14) becomes

$$\begin{aligned} \psi_{n,m}(r, \phi) &= \sqrt{\frac{\gamma^{|\beta|+1} n!}{\pi(n+|\beta|)!}} r^{|\beta|} e^{-\gamma r^2/2} L_n^{(|\beta|)}(\gamma r^2) e^{im\phi} \\ &= \frac{1}{|\beta|!} \sqrt{\frac{\gamma^{|\beta|+1} (n+|\beta|)!}{\pi n!}} r^{|\beta|} e^{-\gamma r^2/2} F(-n, |\beta| + 1; \gamma r^2) e^{im\phi}. \end{aligned} \quad (30)$$

The energy levels in Eq. (25) differ from the usual Landau levels in cylindrical coordinate system [47] to which it transforms when  $\xi = 0$ , and  $a \rightarrow 0$  (i.e., when the chemical potential of dot and antidot vanishes, i.e.,  $V_0 \rightarrow 0$ ). Nevertheless, the Landau levels are nearly continuous discrete spectrum for a particle confined to a large box with  $B=0$  to equally spaced levels corresponding to  $B > 0$ . Each increment of energy,  $\hbar\omega_c$ , corresponding to free particle states, which is the degeneracy of each Landau level leading to a larger spacing as magnetic field  $B$  tends to become stronger [48]. The present model removes this degeneracy with energy levels spectrum becomes

$$E_{n,m} = \hbar\omega_c[n + \frac{1}{2}(|m| + m + 1)], \quad (31)$$

and the wave function reads as

$$\psi_{n,m}(r, \phi) = \frac{1}{m!} \sqrt{\frac{\gamma^{m+1} (n+m)!}{\pi n!}} r^m e^{-\gamma r^2/2} F(-n, m+1; \gamma r^2) e^{im\phi}, \quad (32)$$

where  $\gamma = (\mu\omega_c)/2\hbar$ . In the limit when  $\omega_c \ll g = \sqrt{(8V_0/\mu)c}/r_0$ , then we have

$$E_{nm} = \varepsilon_0 + \varepsilon_1\omega_c + \varepsilon_2\omega_c^2 - \varepsilon_4\omega_c^4 + \dots, \quad (33)$$

where

$$\begin{aligned} \varepsilon_0 &= -2V_0 + N_{nm}g, & \varepsilon_1 &= \frac{\hbar m}{2}, & \varepsilon_2 &= \frac{N_{nm}}{2g}, & \varepsilon_4 &= \frac{N_{nm}}{8g^3}, \\ N_{nm} &= \hbar\left(n + \frac{m+1}{2}\right), & g &= \frac{1}{r_0} \sqrt{\frac{8V_0}{\mu}}. \end{aligned} \quad (34)$$

### 3.2. Interband light absorption coefficient

Expressions (25) and (30), obtained above for charge carriers (electron or hole) energy formula and the corresponding wave function in quantum pseudodot under the influence of external magnetic field and AB flux field, allow to calculate the direct interband light absorption coefficient  $K(\bar{\omega})$  in such system and the threshold frequency of absorption. The light absorption coefficient can be expressed as [11–13,49]

$$\begin{aligned} K(\bar{\omega}) &= N \sum_{n,m,\beta n',m',\beta'} \left| \int \psi_{n,m,\beta}^e(\vec{r}, \phi) \psi_{n',m',\beta'}^h(\vec{r}, \phi) r dr d\phi \right|^2 \\ &\quad \delta(\Delta - E_{n,m,\beta}^e - E_{n',m',\beta'}^h), \\ &= N \sum_{n,m,\beta n',m',\beta'} \frac{\gamma^{|\beta|+|\beta'|+2} (n+|\beta|)! (n'+|\beta'|)!}{\pi^2 n! n'! (|\beta|!)^2 (|\beta'|!)^2} \\ &\quad \left| \int_0^{2\pi} e^{i(m+m')\phi} d\phi \int_0^\infty r dr e^{-(\gamma+\gamma')r^2/2} r^{|\beta|+|\beta'|} \right. \\ &\quad \left. \times F(-n, |\beta| + 1; \gamma r^2) F(-n', |\beta'| + 1; \gamma' r^2) \right|^2 \delta(\Delta - E_{n,m,\beta}^e - E_{n',m',\beta'}^h), \end{aligned} \quad (35)$$

where  $\Delta = \hbar\bar{\omega} - \varepsilon_g$ ,  $\varepsilon_g$  is the width of forbidden energy gap,  $\bar{\omega}$  is the frequency of incident light,  $N$  is a quantity proportional to the square of dipole moment matrix element modulus,  $\psi^{e(h)}$  is the wave function of the electron (hole) and  $E^{e(h)}$  is the corresponding energy of the electron (hole).

Now, we proceed to calculate the light absorption coefficient [48–51]:

$$K(\bar{\omega}) = N \sum_{n,m,\beta n',m',\beta'} P_{n,n'}^\beta Q_{n,n'}^\beta \delta(\Delta - E_{n,m,\beta}^e - E_{n',m',\beta'}^h), \quad (36)$$

where

$$P_{n,n'}^\beta = \frac{1}{(|\beta|!)^4} (\gamma\gamma')^{|\beta|+1} \left(\frac{\gamma+\gamma'}{\gamma-\gamma'}\right)^{2(n+n')} \frac{(n+|\beta|)! (n'+|\beta'|)!}{n! n'!} \quad (37)$$

and

$$Q_{n,n'}^\beta = \left[ |\beta|! \left(\frac{2}{\gamma+\gamma'}\right)^{|\beta|+1} {}_2F_1\left(n, n', |\beta| + 1; -\frac{4\gamma\gamma'}{(\gamma-\gamma')^2}\right) \right]^2. \quad (38)$$

Further, using Eqs. (25) and (35), we find the threshold frequency of absorption as

$$\begin{aligned} \hbar\bar{\omega} &= \varepsilon_g + \frac{\hbar}{2} (2n + \sqrt{(m+\xi)^2 + 2\mu V_0 r_0^2/\hbar^2 + 1}) \\ &\quad \sqrt{\left(\frac{qB}{\mu c}\right)^2 + \frac{8V_0}{\mu r_0^2} + \frac{q\hbar B}{2\mu c} (m+\xi)} \\ &\quad + \frac{\hbar}{2} (2n' + \sqrt{(m+\xi)^2 + 2\mu V_0 r_0^2/\hbar^2 + 1}) \\ &\quad \sqrt{\left(\frac{qB}{\mu' c}\right)^2 + \frac{8V_0}{\mu' r_0^2} + \frac{q\hbar B}{2\mu' c} (m+\xi)} - 4V_0. \end{aligned} \quad (39)$$

where  $\xi = \Phi_{AB}/\Phi_0$  is an integer and  $q = e$ . In the absence of the AB flux field, i.e., when  $\Phi_{AB} = 0$ , we find that the threshold frequency of absorption is identical to Eq. (22) of Ref. [49]. Further, taking  $n = m = 0$ , then we have

$$\begin{aligned} \hbar\bar{\omega}_{00} &= \varepsilon_g + \frac{\hbar}{2} (\sqrt{\xi^2 + 2\mu V_0 r_0^2/\hbar^2 + 1}) \sqrt{\left(\frac{qB}{\mu c}\right)^2 + \frac{8V_0}{\mu r_0^2} + \frac{q\hbar B}{2\mu c} \xi} \\ &\quad + \frac{\hbar}{2} (\sqrt{\xi^2 + 2\mu' V_0 r_0^2/\hbar^2 + 1}) \sqrt{\left(\frac{qB}{\mu' c}\right)^2 + \frac{8V_0}{\mu' r_0^2} + \frac{q\hbar B}{2\mu' c} \xi} - 4V_0. \end{aligned} \quad (40)$$

in the presence of magnetic and AB fields.

For transition  $000 \rightarrow 000$ , the argument of Dirac delta function allows one to define the threshold value of absorption as

$$\frac{\hbar\bar{\omega}_{00}}{\varepsilon_g} = 1 + \frac{(E_0^e + E_0^h)}{\varepsilon_g}, \quad (41)$$

in which for quantum dot, we have

$$\frac{E_0^e}{\varepsilon_g} = \frac{1}{2}(\xi + 1)\sqrt{k^2 + \frac{8}{\rho^2} - \frac{2V_0}{\varepsilon_g}}, \quad (42a)$$

$$\frac{E_0^h}{\varepsilon_g} = \frac{1}{2}(\xi + 1)\sqrt{k'^2 + \frac{8}{\rho'^2} - \frac{2V_0}{\varepsilon_g}}, \quad (42b)$$

$$\rho = \frac{r_0\varepsilon_g}{\hbar}\sqrt{\frac{\mu}{V_0}}, \quad \rho' = \frac{r_0\varepsilon_g}{\hbar}\sqrt{\frac{\mu'}{V_0}},$$

$$k = f(B) = \frac{e\hbar B}{\mu C\varepsilon_g}, \quad k' = f(B) = \frac{e\hbar B}{\mu' C\varepsilon_g}. \quad (42c)$$

However, for quantum antidot, we have

$$\frac{E_0^e}{\varepsilon_g} = \frac{1}{2}(\sqrt{\xi^2 + 2\mu V_0 r_0^2/\hbar^2} + 1)\left(\frac{e\hbar B}{\mu C\varepsilon_g}\right) - \frac{2V_0}{\varepsilon_g}, \quad (43a)$$

$$\frac{E_0^h}{\varepsilon_g} = \frac{1}{2}(\sqrt{\xi^2 + 2\mu' V_0 r_0^2/\hbar^2} + 1)\left(\frac{e\hbar B}{\mu' C\varepsilon_g}\right) - \frac{2V_0}{\varepsilon_g}. \quad (43b)$$

In the absence of the AB flux field, i.e.,  $\xi = 0$ , the above equations (42a) and (42b) become identical to Eq. (27) of Ref. [49] and Ref. [12] in quantum dot. Firstly, we study the variations of the threshold frequency of absorption  $\bar{\omega}_{00}$  (in units of  $\varepsilon_g$ ) as a function of magnetic field (in units of  $k$ ). It is seen that  $\bar{\omega}_{00}$  increases when the applied magnetic field increases (see Fig. 1). The effect of AB flux field on the interband energy is that the lines remain linear but fan out or pushed up along the positive energy when  $\xi$  increases. Secondly, the variations of the threshold frequency of absorption  $\bar{\omega}_{00}$  (in units of  $\varepsilon_g$ ) with quantum dot size (in units of  $\rho$ ). It is seen in Fig. 2 that  $\bar{\omega}_{00}$  decreases when the quantum dot size increases. However, it increases when the

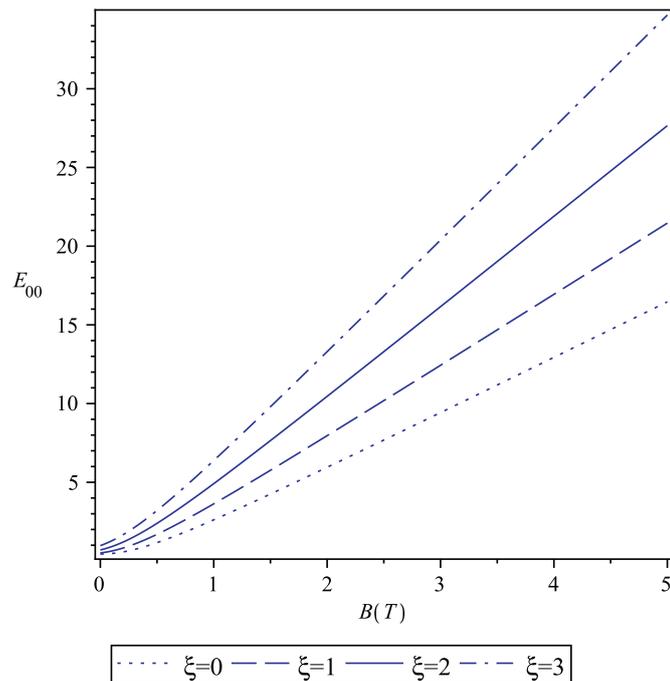


Fig. 1. The ground energy level (in units of meV) as a function of  $B$  for different values of AB flux field.

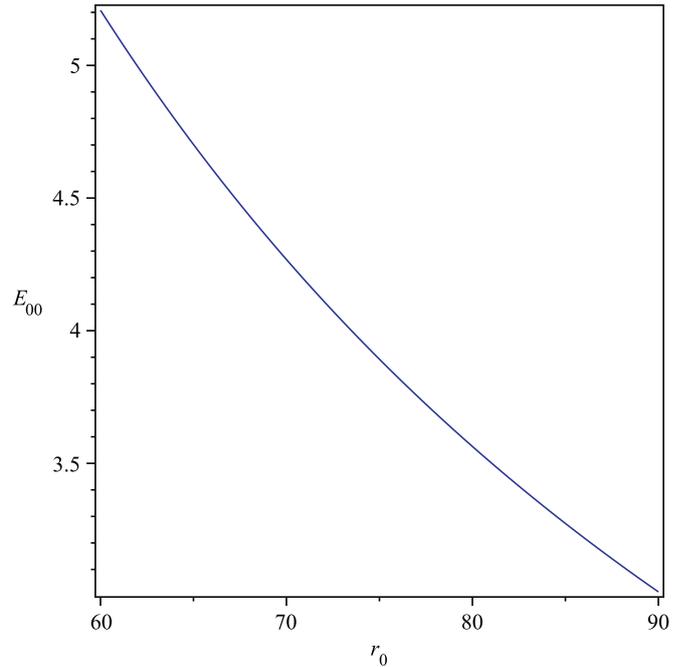


Fig. 2. The ground energy level (in units of  $\varepsilon_g$ ) as a function of quantum dot size  $r_0$  (Å).

quantum pseudodot size increases [12,49]. Furthermore, the variations of  $\bar{\omega}_{00}$  as a function of magnetic field at small (large) applied  $B$  is nonlinear (linear) as shown in Fig. 1 which is in agreement with Ref. [49] when  $\xi = 0$ . Finally, in the presence of AB field, it changes linearly as  $\xi$  increases.

### 3.3. Temperature effect on effective mass and absorption threshold frequency

The variation of the effective mass with temperature is determined according to the expression [32,52,53]

$$\frac{\mu_e}{\mu(T)} = \frac{1}{f(T)} = 1 + E_p^I \left[ \frac{2}{E_g^I(T)} + \frac{1}{E_g^I(T) + \Delta_0} \right], \quad (44)$$

where  $\mu_e$  is the electronic mass,  $E_p^I = 7.51$  eV is the energy related to the momentum matrix element,  $\Delta_0 = 0.341$  eV is the spin-orbit splitting and  $E_g^I(T)$  is the temperature-dependence of the energy gap (in eV units) at the  $\Gamma$  point which is given by [13,52,54,55]

$$E_g^I(T) = 1.519 - \frac{(5.405 \times 10^{-4})T^2}{T + 204} \text{ (eV)}. \quad (45)$$

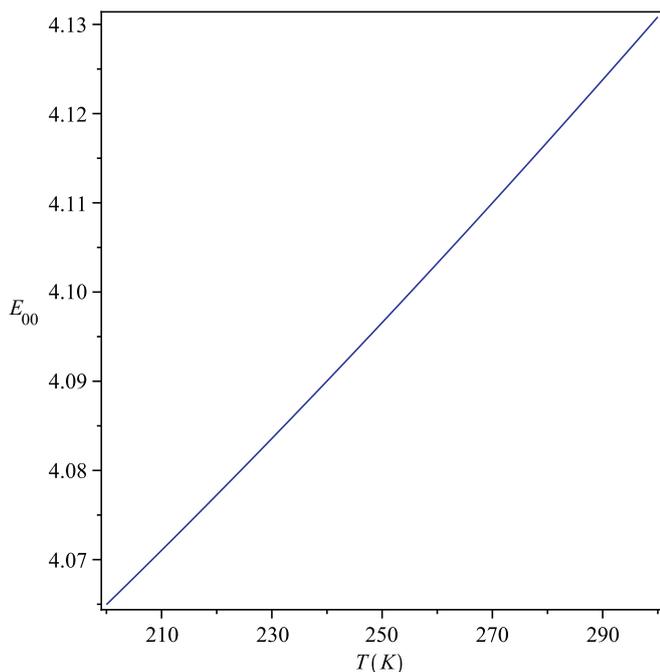
In Table 1, we display the temperature-dependent effective mass to the effective mass of donor electron, i.e.,  $\mu(T)/\mu_e$  for different values of temperatures. As seen in Table 1, the increase in the temperature leading to a decrease in the value of  $f(T) = \mu(T)/\mu_e$ . As a matter of fact, the decrease in this value means that kinetic energy of the donor electron decrease and consequently lowering the binding energy. The results are similar to Ref. [32]. Hence the temperature dependence energy spectrum formula can be expressed as

$$E_{n,m}(B,T) = \frac{\hbar\omega_c}{f(T)} \left[ \sqrt{1 + 4\frac{\omega_D^2}{\omega_c^2}f(T)} \left( n + \frac{\sqrt{(m+\xi)^2 + a^2f(T)+1}}{2} \right) + \frac{m+\xi}{2} \right] - 2V_0, \quad (46)$$

**Table 1**  
Calculated  $f(T)$  with different values of temperature for GaAs.

$T$ (K)	$\mu(T)/\mu_e$
0	0.0669984
10	0.0669886
20	0.0669608
30	0.0669174
40	0.0668603
50	0.0667911
60	0.0667112
70	0.0666217
80	0.0665236
90	0.0664178
100	0.0663051
110	0.0661861
120	0.0660614
130	0.0659315
140	0.0657968
150	0.0656577 <sup>a</sup>
160	0.0655147
170	0.0653679
180	0.0652177
190	0.0650643
200	0.0649080
210	0.0647490
220	0.0645874
230	0.0644235
240	0.0642573
250	0.0640891 <sup>a</sup>
260	0.0639188
270	0.0637468
280	0.0635730
290	0.0633976
300	0.0632206 <sup>a</sup>
350	0.0623154
400	0.0613818 <sup>a</sup>
500	0.0594513 <sup>a</sup>

<sup>a</sup> See Ref. [32].



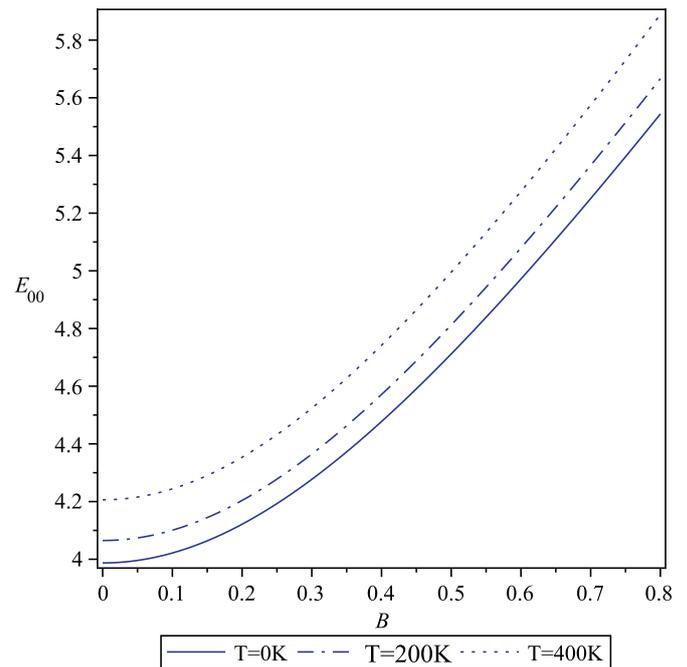
**Fig. 3.** Ground states energy (in meV) as a function of temperature  $T$  in the absence of external fields.

which for GaAs turns to be

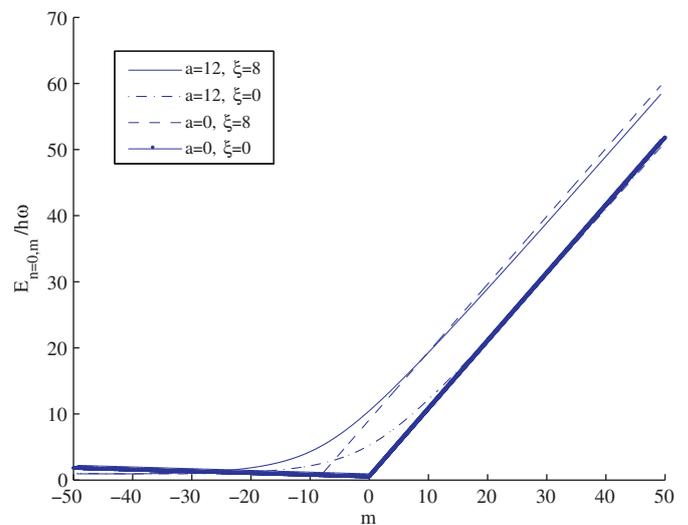
$$E_{n,m}(B,T) = 14.9254\hbar\omega_c \left[ \sqrt{1 + 0.268 \frac{\omega_D^2}{\omega_c^2}} \left( n + \frac{\sqrt{(m+\xi)^2 + 0.067a^2 + 1}}{2} \right) + \frac{m+\xi}{2} \right] - 2V_0, \quad (47)$$

where we have used  $\mu = 0.067\mu_e$ .

To investigate the dependence of the energy levels on temperature, we take the values of parameters:  $B=6$  T,  $\xi=8$ ,  $V_0=0.68346$  meV and  $r_0=8.958 \times 10^{-6}$  cm [25]. Hence, the temperature dependence of the energy levels (in the units of



**Fig. 4.** Ground states energy (in meV) as a function of  $B$  for different temperatures in the absence of AB flux field.



**Fig. 5.** Ground state pseudodot energy level (in units) as a function of magnetic quantum number in the presence and absence of PHQD potential (and) and AB flux field ( $\xi=8$  and  $\xi=0$ ), respectively, when the magnetic field  $B=6$  T.

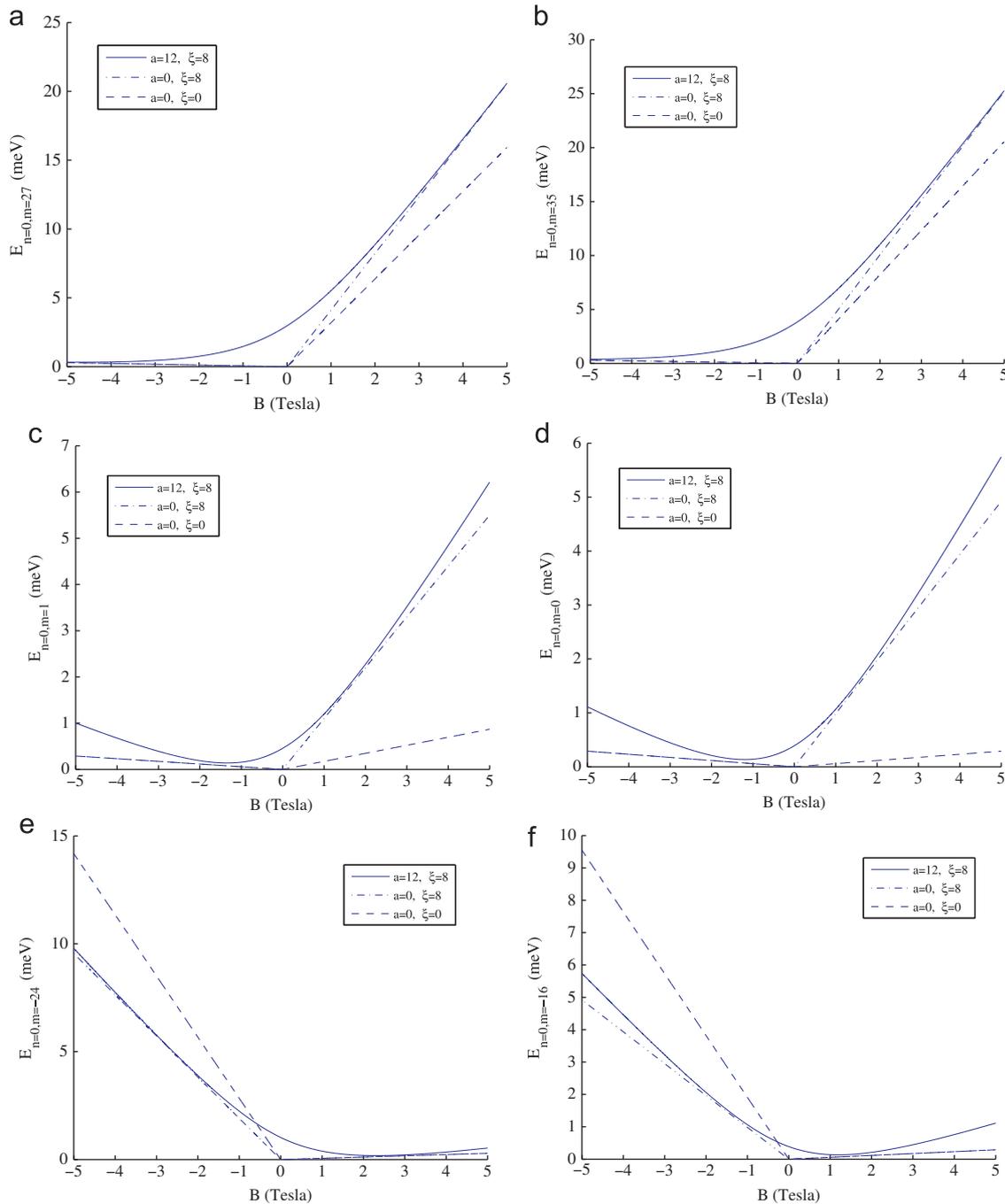
$\hbar\omega_c$ ) at the  $\Gamma$  point are given by

$$\frac{E_{n,m}(T)}{\hbar\omega_c} = \frac{1}{f(T)} \left[ \sqrt{1 + (0.32804)^2 f(T)} \left( n + \frac{\sqrt{(m+8)^2 + 144f(T)} + 1}{2} \right) + \frac{m+8}{2} \right] - 1.9678584, \quad (48)$$

where  $f(T)$  is calculated in Table 1 at any temperature value. In GaAs, we have  $f(T) = 0.067$  [11]. Taking the special values of parameters  $\xi = 8$ ,  $V_0 = 0.68459$  meV and  $r_0 = 8.958 \times 10^{-6}$  cm

[25], two parameters (temperature and magnetic field) dependence of the energy levels (in units of meV) are calculated as

$$E_{n,m}(B,T) = \frac{1}{f(T)} \left[ 0.1157705 \sqrt{B^2 + 3.8803305f(T)} \left( n + \frac{\sqrt{(m+\xi)^2 + 144f(T)} + 1}{2} \right) + 0.1157705B \left( \frac{m+\xi}{2} \right) \right] - 1.36918 \text{ meV}, \quad (49)$$



**Fig. 6.** Ground state p-dot energy level (in meV) as a function of magnetic field  $B$  (in Tesla). The solid, dotted and dashed curves represent the p-dot energy level in the presence of AB flux field, Landau energy level in the presence of AB flux field and Landau level in the absence of AB flux field, respectively, when the magnetic quantum number (a)  $m=27$ , (b)  $m=35$ , (c)  $m=1$ , (d)  $m=0$ , (e)  $m=-24$ , (f)  $m=-16$ .

which becomes

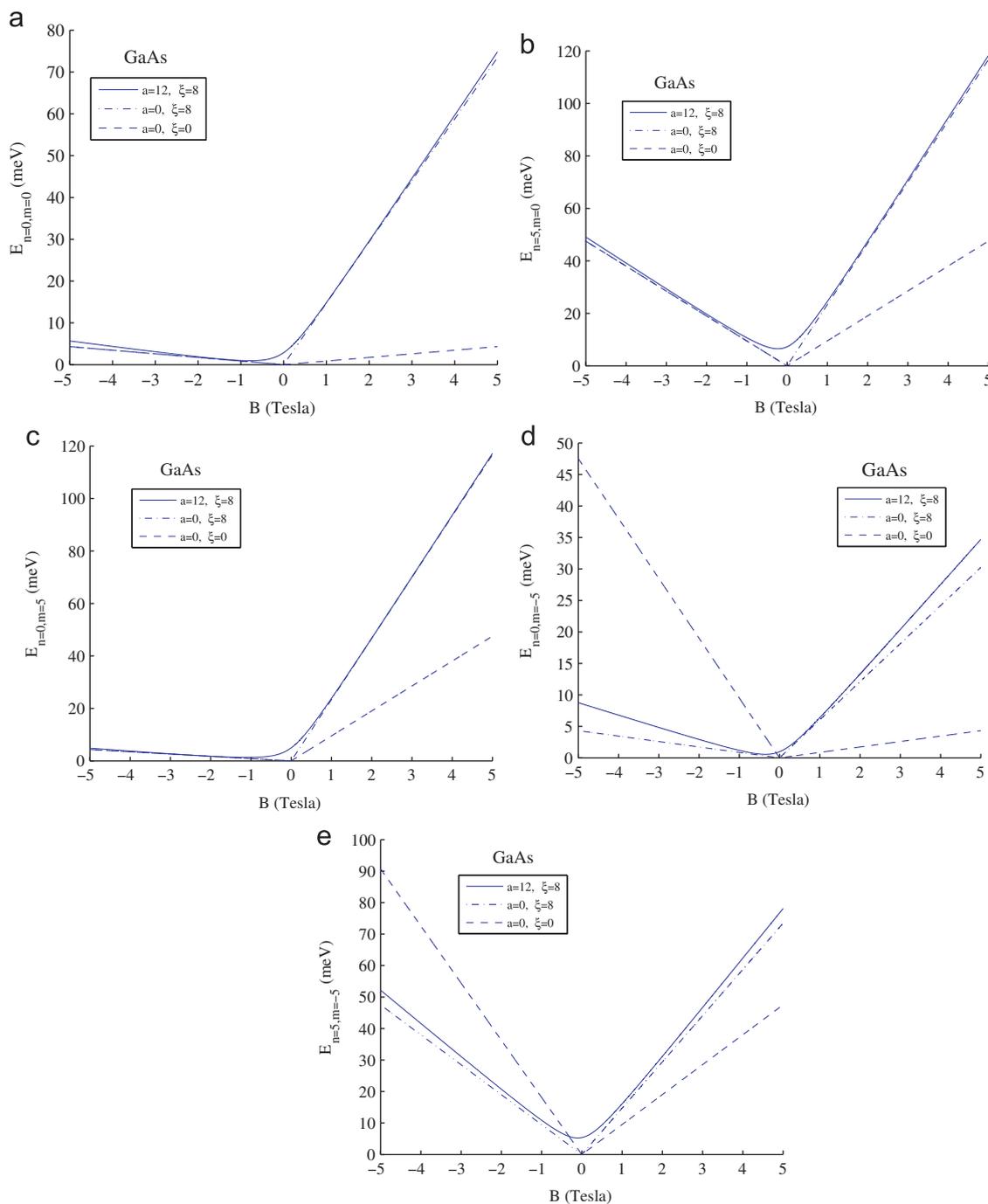
$$E_{n,m}(B) = 14.9254 \left[ 0.1157705 \sqrt{B^2 + 0.26} \left( n + \frac{\sqrt{(m+\zeta)^2 + 9.648} + 1}{2} \right) + 0.1157705B \left( \frac{m+\zeta}{2} \right) \right] - 1.36918 \text{ meV.} \quad (50)$$

When  $n=0$  and  $m=0$ , we obtain

$$E_{00}(B,T) = \frac{1}{2f(T)} [0.1157705(\sqrt{\zeta^2 + 144f(T)} + 1)$$

$$\times \sqrt{B^2 + 3.8803305f(T) + 0.1157705B\zeta}] - 1.36918 \text{ meV.} \quad (51)$$

It is seen in Fig. 3 that the energy of the ground state is linearly changing with temperature for high temperatures when external fields are absent, i.e.,  $\zeta=0$  and  $B=0$ . As seen from Fig. 4, for specific value of temperature, the variation of the ground energy with the magnetic field is linear (nonlinear) for large (small) magnetic field values. Furthermore, the variation of the energy with the temperature increases with the increasing temperature as demonstrated in Table 1 and Fig. 4. For further details see



**Fig. 7.** GaAs pdot energy level (in meV) as a function of magnetic field  $B$  (in Tesla). The solid, dotted and dashed curves represent the pdot energy level in the presence of AB flux field, Landau energy level in the presence of AB flux field and Landau level in the absence of AB flux field, respectively, for (a)  $n=m=0$ , (b)  $n=5$  and  $m=0$ , (c)  $n=0$  and  $m=5$ , (d)  $n=0$  and  $m=-5$ , (e)  $n=5$  and  $m=-5$ .

Figs. 1–5 given in Ref. [4] when we set  $\zeta = 0$  in the above energy formula.

#### 4. Results and discussions

We solved exactly the Schrödinger equation for an electron under the pseudoharmonic interaction consisting of quantum dot potential and antidot potential in the presence of a uniform strong magnetic field  $\vec{B}$  along the z-axis and AB flux field created by an infinitely long solenoid inserted inside the pseudodot. We have obtained bound state solutions including the energy spectrum formula (25) and wave function (30) for a Schrödinger electron. Now we study the effect of the pseudoharmonic potential, the presence and absence of magnetic field  $B$ , the AB flux density  $\zeta$  and the antidot potential on the energy levels (25). To see the dependence of the energy spectrum on the magnetic quantum number,  $m$ , we take the following values: magnetic field  $\vec{B} = (6T)\hat{z}$ , AB flux field  $\zeta = 8$ , chemical potential  $V_0 = 0.68346$  meV and  $r_0 = 8.958 \times 10^{-6}$  cm [25]. Thus, we obtained  $a = \sqrt{2\mu V_0 r_0^2 / \hbar^2} = 11.997702$ ,  $2\omega_D = \sqrt{8V_0 / \mu r_0^2} = 0.3280381\omega_c$  and  $\hbar\omega = 1.05243\hbar\omega_c$  with  $\hbar\omega_c = 0.1157705B$  meV where  $B$  is in Tesla [50], the dependence of the energy on the quantum numbers  $n$  and  $m$  is given by

$$\frac{E_{n,m}}{\hbar\omega_c} = 1.05243 \left( n + \frac{\sqrt{(m+8)^2 + 12^2 + 1}}{2} \right) + \frac{1}{2}(m+8) - 1.9678584 \quad \text{for } B = 6 \text{ T}, \quad (52)$$

where  $m = 0, \pm 1, \pm 2, \dots$  and  $n = 0, 1, 2, \dots$ . For the lowest ground state ( $n=0$ ):  $E_{0,m}/\hbar\omega_c = 1.05243(\sqrt{(m+8)^2 + 12^2 + 1})/2 + (m+8)/2 - 1.9678584$ , for  $B = 6$  T. Overmore, to show the effect of magnetic field  $B$  on the energy spectrum, we take values for parameters  $\zeta = 8$ ,  $V_0 = 0.68459$  meV and  $r_0 = 8.958 \times 10^{-6}$  cm [25], where  $a = \sqrt{2\mu V_0 r_0^2 / \hbar^2} = 12.007617$  and  $4\omega_D^2 = 8V_0 / \mu r_0^2 = 0.120039 \times 10^{24}$  (rad/s)<sup>2</sup>, the dependence of energy levels on the magnetic field becomes

$$E_{n,m}(\text{meV}) = 0.1157705\sqrt{B^2 + 3.8803305} \left( n + \frac{\sqrt{(m+\zeta)^2 + 12^2 + 1}}{2} \right) + 0.1157705B \left( \frac{m+\zeta}{2} \right) - 1.36918, \quad \zeta = 8. \quad (53)$$

Note that in the absence of AB flux field ( $\zeta = 0$ ), Eq. (52) resembles Eq. (13) of Ref. [49].

In Fig. 5, we plot the pseudodot energy levels in the absence (presence) of pseudodot potential (i.e.,  $V_0 = 0 \rightarrow a = 0$  ( $V_0 \neq 0 \rightarrow a = 12$ )) and in the absence (presence) of AB flux field  $\Phi_{AB}$  (i.e.,  $\zeta = 0$  ( $\zeta = 8$ )) as a function of magnetic quantum number  $m$  for  $B = 6$  T. As demonstrated in Fig. 5, the Landau energy states [47] (i.e.,  $V_0 = 0 \rightarrow a = 0$ ,  $\zeta = 0$  and  $\zeta = 8$ ) are degenerate states (see, long dashed and dotted solid curves) for negative values of  $m$ , however, the pseudodot potential removes this degeneracy (case when  $V_0 \neq 0 \rightarrow a = 12$ ), (see, solid and dotted dashed curves). In the absence of pseudodot potential ( $a = 0$ ) and presence of AB flux field ( $\zeta = 8$ ), the degeneracy still exists (long dashed line). It is found that the energy levels of PHQD potential are approximately equal to the Landau energy levels for large absolute  $m$  values. However, they are quite different for small absolute  $m$  values ( $-12 \leq m \leq 13$  when  $\zeta = 0$  and  $-20 \leq m \leq 5$  when  $\zeta = 8$ ). It is also noted that as the quantum number  $n$  increases ( $n > 0$ ), the curves are quite similar to Fig. 5 but the energy levels are pushed up toward the positive energy for all values of  $m$ . In Fig. 6a–f, we plot the magnetic field dependence of the ground state energy  $E_{0,m}(\zeta, a)$  (in units of meV) in the presence and absence of pseudodot potential and AB flux field for several values of magnetic quantum numbers  $m = 27, 35, 1, 0, -24$  and  $-16$ , respectively. It is shown in Fig. 6a–f that pseudodot energy increases with increasing magnetic field strength. Furthermore, in the absence of pseudodot potential, magnetic field in the positive  $z$  direction removes the degeneracy for positive  $m$  values. In these figures, the behavior of pseudodot energy as a function of the magnetic field  $B$  is shown in the presence of pseudodot potential and AB flux field (solid curves), in the absence of pseudodot potential and presence of AB flux field (dotted curves) and the absence of pseudodot potential and AB flux field (dashed curves).

For GaAs, in Fig. 7, we show the variation of the pseudodot energy levels (in meV) as a function of magnetic field  $B$  (in Tesla) (52). We consider the cases: the presence of both pseudodot potential and  $\zeta$  (solid curves), the absence of pseudodot potential and presence of  $\zeta$  (dotted curves) and the absence of both pseudodot potential and  $\zeta$  (dashed curves) taking the various values of radial  $n$  and magnetic  $m$  quantum numbers. For GaAs case, we consider the following cases (a)  $n = m = 0$ , (b)  $n = 5, m = 0$ , (c)  $n = 0, m = 5$ , (d)  $n = 0, m = -5$  and (e)  $n = 5, m = -5$  in Fig. 7a–e, respectively. In Fig. 8, we plot the energy levels (52) for various values of  $n$  and  $m$  quantum numbers as functions of the magnetic field strength  $B$  for the case  $\zeta = 0$ . It is seen that the

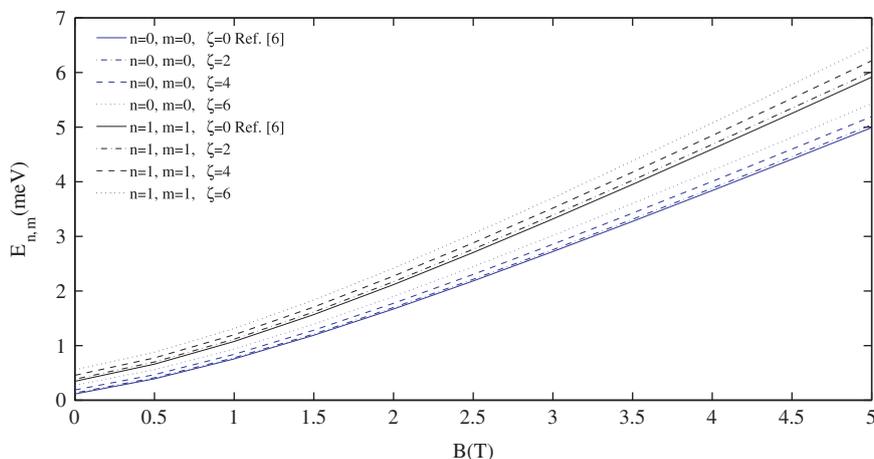


Fig. 8. The energy levels for various values of  $n$  and  $m$  quantum numbers as functions of the magnetic field strength  $B$  for the case  $\zeta = 0$  and  $\zeta \neq 0$ .

energy curves coincide with those obtained by Eq. (13) of Ref. [49]. We also plot the case where  $\xi \neq 0$ .

## 5. Concluding remarks

In this work, we have obtained bound state energy levels and wave functions of the Schrödinger particle in the 2D pseudoharmonic quantum dot and antidot structure under the influence of external uniform magnetic and AB flux fields. Overmore, the Schrödinger bound state solutions are obtained, in closed form, in the framework of the NU method. In our application, we have calculated the energy and wave function solutions for a few electrons bound at GaAs semiconductor interfaces whose velocities are no relativistic. Overmore, the nonrelativistic electron and hole energy spectra and the corresponding wave functions are used to calculate the interband light absorption coefficient and the threshold frequency of absorption. This energy spectrum of the electron (hole) may be also used in studying the thermodynamics properties of quantum structures with dot (antidot) for specific values of external uniform magnetic and AB flux fields and spatial confinement length. The temperature dependence of the energy levels are calculated in Table 1 at any temperature  $T$  (Kelvin).

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