Momentum Conservation in the Aharonov-Casher Effect (*).

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Summary. — In the Aharonov-Casher effect, the neutron exerts a non-vanishing force on a static line charge (a charged wire). The reaction of this force is the time rate of change of the electromagnetic momentum.

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1. – Introduction.

The Aharonov-Casher (AC) effect [1] has been the topic of much discussion over the past ten years. AC showed that neutrons moving in a plane perpendicular to a line charge and passing it on opposite sides will experience no force but a relative quantum phase shift occurs. Experimental and theoretical treatment of this effect has been carried out [2-5].

The purpose of this paper is to point out that the moving neutron does exert a force on the nearby line of charge and the reaction of this force is the time rate of change of the electromagnetic momentum. Three different momenta are involved in the interaction: that of the neutron, the line of charge, and the electromagnetic field.

2. – Force exerted on the line of charge.

Consider the problem of a moving neutron in a static electric field. The source of this field is a line charge (a charged wire), which is assumed to be rigid and fixed in spatial orientation. The magnetic moment of the neutron is parallel to the line charge. Let $\boldsymbol{r}''$ be the displacement of a fixed point in the body of the source. Let $\boldsymbol{r}$ be the displacement of the neutron. The interaction energy of the charge source and the

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neutron is

$$\Delta E = \frac{1}{4\pi}\int \boldsymbol{E}_{\rm n}(\boldsymbol{r}'-\boldsymbol{r})\cdot\boldsymbol{E}_{\rm ext}(\boldsymbol{r}'-\boldsymbol{r}'')\,{\rm d}^3\boldsymbol{r}', \tag{1}$$

where $\boldsymbol{E}_{\rm n}$ is the electric field of the neutron, and $\boldsymbol{E}_{\rm ext}$ is the static electric field of the line charge.

The force on the line charge is

$$\boldsymbol{F} = \frac{1}{4\pi}\boldsymbol{\nabla}''\int \boldsymbol{E}_{\rm n}(\boldsymbol{r}'-\boldsymbol{r})\cdot\boldsymbol{E}_{\rm ext}(\boldsymbol{r}'-\boldsymbol{r}'')\,{\rm d}^3 r', \tag{2}$$

In the non-relativistic limit, the electric field of the neutron in the laboratory frame is

$$\boldsymbol{E}_{\rm n}(\boldsymbol{r}'-\boldsymbol{r}) = -(\boldsymbol{V}/C)\times\boldsymbol{B}_{\rm n}(\boldsymbol{r}'-\boldsymbol{r}), \tag{3}$$

where $\boldsymbol{V}$ is the velocity of the neutron in the laboratory frame and $\boldsymbol{B}_{\rm n}(\boldsymbol{r}'-\boldsymbol{r})$ is the magnetic field of the neutron at the point $\boldsymbol{r}'$. The substitution of eq. (3) into eq. (4) yields

$$\begin{aligned}\boldsymbol{F} &= -\frac{1}{4\pi C}\boldsymbol{\nabla}''\int \boldsymbol{V}\times\boldsymbol{B}_{\rm n}(\boldsymbol{r}'-\boldsymbol{r})\cdot\boldsymbol{E}_{\rm ext}(\boldsymbol{r}'-\boldsymbol{r}'')\,{\rm d}^3 r' = \\ &= -\frac{1}{4\pi C}\boldsymbol{\nabla}''\boldsymbol{V}\cdot\int \boldsymbol{B}_{\rm n}(\boldsymbol{r}'-\boldsymbol{r})\times\boldsymbol{E}_{\rm ext}(\boldsymbol{r}'-\boldsymbol{r}'')\,{\rm d}^3 r' = \\ &= \boldsymbol{\nabla}''[\boldsymbol{V}\cdot\boldsymbol{P}_{\rm field}(\boldsymbol{r}-\boldsymbol{r}'')] = \boldsymbol{\nabla}''[\boldsymbol{V}\cdot\boldsymbol{P}_{\rm field}(\boldsymbol{r}-\boldsymbol{r}'')] = \\ &= -[(\boldsymbol{V}\cdot\boldsymbol{\nabla})\boldsymbol{P}_{\rm field} + \boldsymbol{V}\times(\boldsymbol{\nabla}\times\boldsymbol{P}_{\rm field})] = -[({\rm d}/{\rm d}t)\boldsymbol{P}_{\rm field} + \boldsymbol{V}\times(\boldsymbol{\nabla}\times\boldsymbol{P}_{\rm field})].\end{aligned} \tag{4}$$

Here we have made use of the expression for the electromagnetic momentum, which depends upon the magnetic field of the neutron and the electric field of the charged wire.

Now we need to evaluate $\boldsymbol{\nabla}\times\boldsymbol{P}_{\rm field}$ in the second term on the right-hand side of eq. (4). This can be done as follows: The momentum of the electromagnetic field in the rest frame of the wire is [6]

$$\boldsymbol{P}_{\rm field} = (1/c)\boldsymbol{E}_{\rm ext}\times\boldsymbol{\mu}, \tag{5}$$

where $\boldsymbol{\mu}$ is the magnetic-dipole moment of the neutron. Thus

$$\boldsymbol{\nabla}\times\boldsymbol{P}_{\rm field} = (1/c)\boldsymbol{\nabla}\times\boldsymbol{E}_{\rm ext}\times\boldsymbol{\mu} = (1/c)[\boldsymbol{\mu}\cdot\boldsymbol{\nabla}]\boldsymbol{E}_{\rm ext}, \tag{6}$$

Since the magnetic-dipole moment, $\boldsymbol{\mu}$, is parallel to the line charge (say the z-axis) and $\boldsymbol{E}_{\rm ext}$ is in the (x, y)-plane, it follows that the right-hand side of eq. (6) vanishes. Therefore we have

$$\boldsymbol{\nabla}\times\boldsymbol{P}_{\rm field} = 0, \tag{7}$$

and thus eq. (4) reads

$$F + (\mathrm{d}/\mathrm{d}t)\boldsymbol{P}_{\text{field}} = 0\,. \tag{8}$$

We see that the mechanical force vanishes in the AC effect since $\boldsymbol{V} \times (\boldsymbol{\nabla} \times \boldsymbol{P}_{\text{field}})$ vanishes identically. A similar result is obtained in the Aharnonov-Bohm (AB) effect [7]. There, a recent result of Zhu and Henneberger [8] shows that $\boldsymbol{V} \times \boldsymbol{\nabla} \times \boldsymbol{P}_{\text{field}} = 0$ because $\boldsymbol{\nabla} \times \boldsymbol{P}_{\text{field}} = (e/c)\boldsymbol{B}_{\text{ext}}$ and $\boldsymbol{B}_{\text{ext}} = 0$ in the region accessible to the electron.

Therefore, eq. (8) shows that in the AC effect the force exerted on the line charge is just the negative of the time rate of change of the electromagnetic momentum. If no other forces act on the line charge, then the total momentum of the field plus that of the line charge is conserved. Therefore, the forces on the field and on the line charge constitute an action-reaction pair.

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