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Numerical analysis to solve the hydraulics of trickle irrigation units

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Abstract. A model to solve the hydraulics of trickle irrigation units is developed in this study. This model is based on utilizing Newton Raphson technique. The model converts laterals into equivalent outlets through utilizing a simple power relation between inlet lateral discharge and hydraulic head. This relation is obtained through least squares analysis between inlet lateral discharge and hydraulic head. This study showed that this relation with only two coefficients is sufficient to describe the relation between inlet lateral discharge and hydraulic head. Based on this relation, the model converts manifold lines into equivalent laterals and solves their hydraulics by Newton Raphson technique. After that solution, the model evaluates trickle irrigation units by estimating statistical uniformity and Christiansen uniformity coefficients and checks the solution obtained through forward step method for each lateral. Several numerical examples for utilizing the model are presented in this paper.

Key words: hydraulics, laterals, manifold, trickle irrigation

Abbreviations: C_H(j) – Hazen-Williams coefficient for lateral segment # j; C(j) – lateral line coefficient for segment #j; d(j) – Diameter of lateral segment # j in mm; E(j) – Elevation of outlet # j; $\mathbf{f}'(\mathbf{H_m})$ – First derivative of **f(H)** evaluated at $\mathbf{H}_m$; H(j) – Total hydraulic head at outlet # j; $\mathbf{H}_m$ – Hydraulic head vector (**H**) as determined (or assumed) from iteration # m; $\mathbf{H}_{m+1}$ – Improved estimate of vector **H** for the following iteration (m+1); H_o – Inlet pressure at inlet point of the lateral/manifold line; h_f(j) – Head loss in lateral segment number j; K&x – Outlet pressure-discharge coefficients; L(j) – Length of lateral segment j in meters; q – Outlet discharge; q(j) – Discharge from outlet number j; Q(j) – Flow rate in lateral segment number j

Introduction

Hydraulic analysis of trickle irrigation units is based on the hydraulics of pipelines with multiple outlets. Well known, Christiansen's *F* factor was introduced to estimate the friction head losses along a pipe with multiple outlets, equally spaced with constant discharge (Christiansen 1942). Wu & Gitlin (1974) introduced a method to describe the pressure distribution along a lateral line assuming that discharge is uniformly distributed along that line. As the distance between the first outlet and the beginning of the lateral line is some times equal to half the spacing between other outlets, Christiansen's

F factor was adjusted for that situation. Scaloppi (1988) adjusted *F* factor to compute pressure head loss in pipes having multiple, equally spaced outlets for any given distance from the first outlet to the beginning of the pipe/lateral. Anwar (1999) introduced a *G* factor to consider outflow from the downstream end of a lateral line. This allowed the application of this correction factor to calculate friction losses in tapered laterals. Analytical equations for two average pressure correction factors for linear displacement laterals with or without outflow at the downstream end were also developed (Anwar 2000a). These factors were adjusted to be used when the first outlet is a fraction of a full spacing from the lateral inlet (Anwar 2000b).

The analytical solutions mentioned above assume constant discharge from outlets along laterals. However, discharge from outlets is function of pressure head along lateral lines unless pressure compensating outlets are utilized. Warrick & Yitayew (1988) presented an alternative treatment that includes a spatially variable discharge function as part of the basic solution. This approach was utilized for deriving an analytical solution of trickle irrigation hydraulics for the design of laterals (Yitayew & Warrick 1988). Valiantzas (1998) presented an analytical approach to improve the accuracy of previous analytical approaches through assuming a varied flow along the lateral (power relation).

The other approach to solving the hydraulics of lateral lines is numerical. Numerical approaches solve the problem either backward or forward and can take into consideration the variability in discharge, pressure, diameter, spacing, etc. Numerical approaches became popular with the development of personal computers. These approaches to solve the hydraulics of trickle systems included the use of finite elements (Bralts & Segerlind 1985; Kang & Nishiyama 1996a; Kang & Nishiyama 1996b).

Solving the hydraulics of trickle irrigation systems requires solving sets of nonlinear equations which is common in solving the hydraulics of pipe networks. There are two main approaches in solving these systems of equations. The first approach is a successive linear approximation method in which these equations are linearized using an initial solution. This results in converting the system of nonlinear equations into a set of linear equations. Solving such sets is quite common in the finite element method where symmetric banded matrices are solved efficiently. The results are used as an improved estimate of an initial solution; then a new system of linear equations is formed and solved again. The procedure is continued until convergence (Jeppson 1977). The successive linear approximation approach was implemented to solve the hydraulics of trickle systems (Hathoot et al. 1993; Bralts & Segerlind 1985; Kang & Nishiyama 1996a, 1996b).

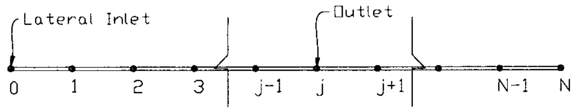

Figure 1. A trickle line with N outlets.

The second approach is to use Newton Raphson method to solve the system of nonlinear equations. This method showed a speed of convergence much faster than successive linear approximation when the two methods start from the same initial solution in analyzing a set of small hypothetical trickle irrigation systems (Mizyed 1997). However, the biggest disadvantage of applying both successive linear approximation and Newton Raphson methods to trickle irrigation units is memory requirements. This results from considering all laterals and outlets in a real size trickle irrigation unit. This paper utilizes Newton Raphson method in analyzing trickle irrigation units and it considers alternatives to overcome memory problems.

Hydraulics of trickle irrigation lines

Considering a trickle lateral line with N outlets, these outlets are numbered from 1 to N along a lateral line (Figure 1). Outlet number 1 is connected to the manifold or to pressure regulator with known head (H_o). Discharge from each outlet is given by (Walker, 1980):

$$q = K(H - E)^x \tag{1}$$

Where:

q : Outlet discharge

H : Total hydraulic head at outlets

E : Elevation of outlet

K and x are outlet pressure-discharge coefficients.

The difference in head between any two successive outlets is equal to the friction head loss in the lateral line segment connecting these outlets. Friction head loss could be taken from Hazen-Williams formula, which is (Jeppson 1977):

$$h_f(j) = C(j)^* Q^{1.85}(j) \tag{2}$$

Where:

Q(j) : Flow rate in lateral line segment number j,

h_f(j) : Head loss in lateral line segment number j,

C(j) : Lateral line coefficient given by:

$$C(j) = \frac{1.21^{*}10^{10*}L(j)}{C_H^{1.85}(j)^{*}d^{4.872}(j)} \tag{3}$$

Where:

L(j) : Length of lateral line segment j in meters,

d(j) : Diameter of lateral line segment j in mm,

C_H(j) : Hazen-Williams coefficient for lateral line segment j.

To determine flow rate in a lateral (pipe) segment from head loss along that segment, Eq. (2) can be written as:

$$Q(j) = \left(\frac{h_f(j)}{C(j)}\right)^{0.54} \tag{4}$$

The head loss along the lateral segment can be written as the difference in head between the two nodes connected by the lateral segment. Considering any two successive nodes j–1 and j in the lateral line, the following equation gives flow rate in lateral segment #j:

$$Q(j) = \left(\frac{H(j-1) - H(j)}{C(j)}\right)^{0.54} \tag{5}$$

To solve the hydraulics of a trickle line, the principle of continuity should be satisfied at each outlet (node) in the trickle line. Considering any node (# j) along a lateral line, continuity principle requires that:

$$Q(j) = Q(j+1) + q(j) \tag{6}$$

Where:

Q(j) : discharge in trickle line from node j–1 to j

Q(j+1) : discharge in trickle line from node j to j+1.

q(j) : discharge from outlet j given by eq. 1.

Writing equation 6, in terms of total head at nodes results as:

$$\left(\frac{H(j-1)-H(j)}{C(j)}\right)^{0.54} = \left(\frac{H(j)-H(j+1)}{C(j+1)}\right)^{0.54} + K^{*}(H(j)-E(j))^{x} \tag{7}$$

For laterals with equally spaced outlets and constant diameter (a common case in irrigation systems) the coefficients C(j) and C(j+1) are equal. However, different lengths and diameters, different values of C(j) could be used in such a numerical method. For the last outlet (#N) in the lateral, continuity principle gives:

$$\left(\frac{H(N-1)-H(N)}{C(N)}\right)^{0.54} = K^{*}(H(N)-E(N))^{x} \tag{8}$$

For the first outlet in the lateral, continuity principle results as:

$$\left(\frac{H_o-H(1)}{C(1)}\right)^{0.54} = \left(\frac{H(1)-H(2)}{C(2)}\right)^{0.54} + K^{*}(H(1)-E(1))^{x} \tag{9}$$

Where, H_o is pressure at the inlet to the lateral line which might be regulated or controlled by a valve, and/or other hydraulic characteristics of the supply system.

Assuming the inlet pressure H_o is given, then solving the hydraulics of the above trickle line requires solving a set of N nonlinear equations (equations 7 through 9). The above set of N nonlinear equations has a set of N unknowns. This set of nonlinear equations could be solved iteratively through two common methods. The first method is successive linear approximations of these equations and solving the resulting set of linear equations each time through common linear algebra methods (Kang & Nishiyama 1996a). The second method is using Newton Raphson technique which is discussed in the following section. When the set is solved for the unknown values of heads at outlets, outlet discharge could be determined from outlet pressure-discharge equation (eq. 1). After that any performance parameter could be determined for the lateral line.

Newton Raphson technique

Newton Raphson technique is an iterative procedure which starts from an initial solution for the vector of unknowns **H** and this vector is improved gradually until convergence. The set of equations is first converted into the form of **f(H)=0** through moving all non zero terms to the left side of the equation. The solution is obtained utilizing the formula:

$$\mathbf{H}_{m+1} = \mathbf{H}_m - \mathbf{f}(\mathbf{H}_m)/\mathbf{f}'(H_m) \tag{10}$$

Where:

$\mathbf{H}_m$: Vector **H** of Hydraulic head as determined (or assumed) from iteration # m,

$\mathbf{H}_{m+1}$: Improved estimate of vector **H** for the following iteration (m+1),

$\mathbf{f}'(\mathbf{H}_m)$: First derivative of **f(H)** evaluated at $\mathbf{H}_m$.

The above formula is written as:

$$\mathbf{H}_{m+1} = \mathbf{H}_m - \mathbf{D}^{-1}\mathbf{f}(\mathbf{H}_m) \tag{11}$$

where **D** is the Jacobian matrix consisting of the derivative elements. For a set of N equations above, the Jacobian matrix will have a dimension of NxN. However, this Jacobian matrix is symmetric and there are many zeros in it. As the numbers utilized in numbering the outlets are in order, the Jacobian matrix will have a diagonal row and another two symmetric diagonal rows (one above and one below). Utilizing the symmetry and sparse nature of such matrices for trickle laterals results in a matrix with N rows and 2 columns, or:

$$\mathbf{D} = \begin{pmatrix} \frac{\partial f_1}{\partial H(1)} & \frac{\partial f_1}{\partial H(2)} \\ \frac{\partial f_2}{\partial H(2)} & \frac{\partial f_2}{\partial H(3)} \\ \cdots & \cdots \\ \frac{\partial f_j}{\partial H(j)} & \frac{\partial f_j}{\partial H(j+1)} \\ \cdots & \cdots \\ \frac{\partial f_N}{\partial H(N)} & 0 \end{pmatrix} \tag{12}$$

The iterative formula could be written as:

$$\mathbf{H}_{m+1} = \mathbf{H}_m - \mathbf{Z} \tag{13}$$

Where:

$$\begin{aligned} &\mathbf{Z} = \mathbf{D}^{-1*}\mathbf{f}(\mathbf{H}_m) \\ &\mathbf{D}^*\mathbf{Z} = \mathbf{f}(\mathbf{H}_m) \end{aligned} \tag{14}$$

Solving a system of equations shown above (13 and 14) is common in finite elements using some linear algebra computer subroutines. Utilizing the symmetric sparse nature of matrix **D** is essential in reducing computer memory requirements. The system could be solved without utilizing that nature through direct estimation of the inverse of initial matrices and multiplying them by $\mathbf{f}(\mathbf{H}_m)$ or any other method of linear algebra.

Applying the above iterative method to a node on the lateral line such as node #j, requires first rewriting eq. 7 as (assuming equally spaced outlets and constant diameter of the lateral line or C(j)= C(j+1)):

$$f_j = -\left(\frac{H(j-1) - H(j)}{C(j)}\right)^{0.54} + \left(\frac{H(j) - H(j+1)}{C(j)}\right)^{0.54} + K(H(j) - E(j))^x \tag{15}$$

Taking the derivatives of eq. 15 gives:

$$\frac{\partial F_j}{\partial H(j+1)} = -\frac{0.54}{C(j)}\left(\frac{H(j)-H(j+1)}{C(j)}\right)^{-0.46} \qquad (16)$$

$$\frac{\partial f_j}{\partial H(j)} = -\frac{\partial f_j}{\partial H(j-1)} - \frac{\partial f_j}{\partial H(j+1)} + K^{*}x^{*}(H(j)-E(j))^{x-1} \qquad (17)$$

The above procedure could be utilized to improve the estimates of **H** and the procedure continues until convergence or when the difference between $\mathbf{H_m}$ and $\mathbf{H_{m+1}}$ becomes insignificant.

Verification of Newton Raphson model for solving lateral lines

The procedure mentioned above was converted through a FORTRAN code into a model to analyze the hydraulics of trickle irrigation lines. A set of lateral lines shown in Table 1 was used to verify the model. These numerical examples are examples of trickle and sprinkler lines with outlets ranging from 10 to 200. The model determines pressure head distribution, discharge from each outlet and inlet discharge for a lateral line with known characteristics and known inlet pressure heads. It also estimates average discharge, statistical uniformity and Christiansen uniformity coefficients of the lateral line under consideration. Statistical uniformity is defined as:

$$S_U = 100^{*}(1 - C_V) \qquad (18)$$

Where: S_U: Statistical uniformity,

C_V : Coefficient of outlet discharge variations.

Laterals in Table 1 were also analyzed using a backward step solution (distal outlet method) utilizing the head at the last outlet determined by the lateral computer model and the geometric and discharge characteristics of the trickle line. Results of distal outlet method were compared to those estimated by Newton Raphson technique in the computer model.

Table 2 shows that solutions obtained by Newton Raphson technique are similar to distal outlet method. Both solutions obtained the same inlet discharge and the same pressure distribution along lateral lines. Inlet pressure calculated by distal outlet was the same as that used in obtaining the solution by Newton Raphson method. This verifies the model developed even for systems with low uniformity and high discharge variability. The use of distal

Table 1. Description of a set of Numerical examples for lateral lines used in verification of lateral line model.

Lateral	1	2	3	4	5
Number of outlets	100	200	10	20	20
Outlet Spacing (m)	0.5	0.5	10.00	10.0	5.0
Outlet Coefficient(L/sec/m^x)	0.001	0.001	0.05	0.05	0.01
Outlet Exponent	0.5	0.5	0.5	0.5	0.5
Diameter (mm)	16.0	16.0	50.0	50.0	20.0
Hazen-Williams	140.0	140.0	140.0	140.0	140.0
Inlet head (m)	20.0	20.0	30.0	30.0	20.0
Slope (a=0%,b=5%,c=–5%)	a,b,c	a,b,c	a,b,c	a,b,c	a,b,c

outlet is restricted as pressure head is regulated at the inlet point not at distal end of laterals.

Solving a trickle irrigation unit

Considering a trickle irrigation unit with M laterals on both sides of the manifold and assuming that the number of outlets on the right and left sides of the manifold are N1 and N2 respectively, then the total number of outlets in the system will be M∗(N1+N2). The total number of nodes (points of unknown head) in the system will be M∗(N1+N2+1). This will result in increasing the band width of the Jacobian matrix to become N1+N2+1 which is a significant increase in the memory requirement for solving the unit. Therefore, considering all outlets in a trickle unit requires a lot of memory and large sizes of matrices even if the sparse symmetric nature of matrices is utilized. To overcome computer memory requirements, Kang & Nishiyama (1996b) replaced lateral lines by equivalent outlets by converting manifold lines to lateral lines. This enabled using the same methodology utilized in trickle laterals to solve hydraulics of manifold lines and thus trickle irrigation units.

Outlet equivalents of laterals

The first relation to represent laterals is a polynomial equation between inlet discharge and inlet head. The relation is shown as (Kang & Nishiyama 1996b):

$$Q(j) = C_1 + C_2^* H(j) + C_3^* H(j)^2 + C_4^* H(j)^3 + \ldots + C_n^* H(j)^n \quad (19)$$

Table 2. Results of Newton Raphson method as compared to distal outlet method.

Lateral	Newton Raphson method			Backward Step solution (Distal outlet method)			Christiansen uniformity[2] (%)	Statistical uniformity[2](%) (%)
	Inlet head[1] (m)	Inlet discharge[1] (l/s)	Head at last outlet[2] (m)	Head at last outlet[1] (m)	Inlet discharge[2] (l/s)	Inlet head[2] (m)		
1a	20.0	0.399	14.57	14.57	0.399	20.000	96.08	95.31
1b	20.0	0.386	14.98	14.98	0.386	19.999	94.18	93.14
1c	20.0	0.411	14.16	14.16	0.411	19.999	97.65	97.05
2a	20.0	0.542	4.20	4.20	0.542	19.995	78.13	74.07
2b	20.0	0.341	12.57	12.57	0.341	19.998	87.87	85.78
2c	20.0	0.386	9.84	9.84	0.386	19.998	95.86	94.72
3a	30.0	2.677	28.32	28.32	2.678	30.002	93.96	92.45
3b	30.0	2.552	28.49	28.49	2.552	30.002	97.05	96.41
3c	30.0	2.798	28.15	28.15	2.798	30.001	98.59	98.28
4a	30.0	4.806	21.04	21.04	4.806	30.002	95.82	94.89
4b	30.0	4.359	22.92	22.92	4.395	30.001	89.76	87.78
4c	30.0	5.202	19.20	19.20	5.203	30.002	98.24	97.92
5a	20.0	0.692	9.73	9.73	0.692	19.999	91.07	89.14
5b	20.0	0.645	11.40	11.40	0.645	19.999	86.00	83.20
5c	20.0	0.734	8.09	8.09	0.734	19.999	94.70	93.18

[1]given, [2]computed.

Where:

Q(j)	:	inlet discharge for lateral j,
C_1 , $C_2 \ldots C_n$	:	regression coefficients, and
H(j)	:	inlet head for lateral j.

Values of head are assumed at the distal end of laterals and their corresponding values for inlet head and discharge are estimated using distal outlet method considering the geometry and characteristics of the laterals in the system. The set of values for inlet head (H(j)) and corresponding inlet discharge (Q(j)) could be then analyzed using least squares method to determine the regression coefficients (C_1 , $C_2 \ldots C_n$). Regression analysis in this study showed that seven coefficients are sufficient to represent the laterals considered in many numerical examples including those shown in Table 3.

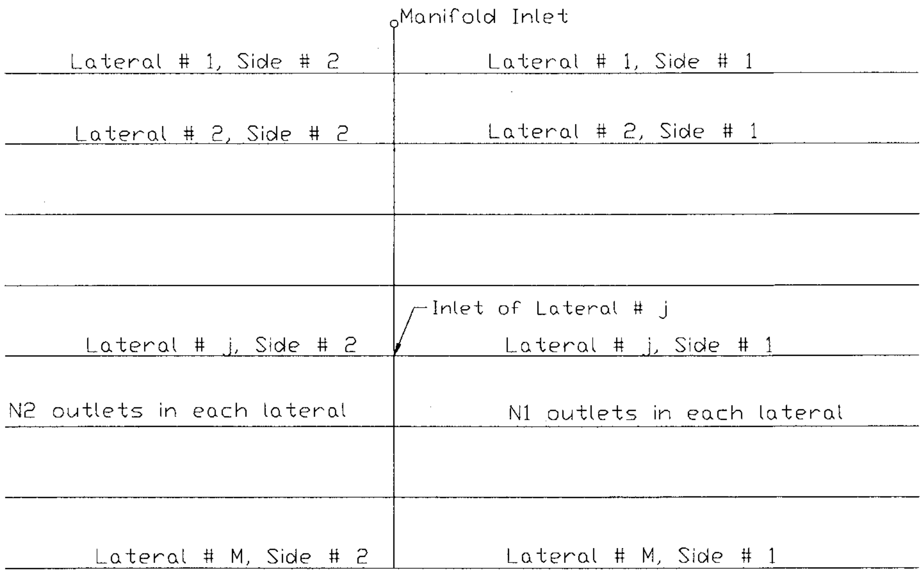

Figure 2. An example of a trickle irrigation unit.

A second relation was investigated which is similar to outlet head-discharge relation shown in eq. 1, or:

$$Q(j) = K\ H(j)^x \tag{20}$$

Where: K and X are regression coefficients.

Equation 20 reduces the relation between inlet discharge and inlet head to a simple relation with only two regression coefficients (K, X). These coefficients are also determined by least squares analysis for inlet discharge versus inlet head.

To investigate the two methods above, laterals shown in Table 1 were considered for numerical examples. For each lateral, polynomial and power relation coefficients in equations 19 and 20 were estimated using least squares analysis. To measure the validity of these relations for each inlet head, the corresponding inlet discharge was estimated using equations 19 and 20. This estimated discharge was compared to the initial discharge estimated earlier by distal outlet. The difference between both is estimation error. The absolute value of the maximum difference for the set is the maximum absolute error of estimation. For each set, the errors were squared and the sum of squared errors was estimated. The sum of squared errors was divided by the number of points in the set to estimate average squared error. Then the square root of the average squared error (SASE) was estimated and shown in Table 3 for both relations. Table 3 shows that both polynomial and power relations could be used for that representation with acceptable error. The polynomial relation produced smaller errors than the power relation in most of the cases. This is due to utilizing 7 coefficients in a polynomial equation compared to two in a power relation. However, the errors estimated by power relation are also small especially when the laterals have small slopes. The maximum errors were observed when the maximum difference in elevation was more than 20% of operating pressure which is usually considered high in irrigation systems. Further numerical examples showed that the accuracy of power relation is significantly improved when the maximum difference in elevation is reduced to 20% or less of operating pressure. In most trickle laterals pressure difference is maintained at about 10% or less of operating pressure. This requires maximum difference in elevation below 10% of operating pressure which improves the accuracy of the power relation equation significantly.

Hydraulic analysis of trickle units/manifolds

After converting lateral lines into equivalent outlets, manifold lines are converted into laterals. If the manifold has laterals on both sides, then there will be two equivalent outlets at each node described by K_1, X_1, K_2 and X_2. This makes the analysis of manifolds similar to that of laterals and thus the same Newton Raphson technique is utilized in the analysis. Therefore, a computer model was developed with analysis summarized as:

1. Convert laterals on both sides of manifold into equivalent outlets, which requires estimating coefficients of power relation between inlet discharge and head shown in equation 20 for laterals on both sides.
2. The manifold is then converted into a lateral line. This line is analyzed by the same model used earlier to analyze lateral lines which utilizes Newton Raphson technique. The result of analysis will give hydraulic distribution and discharge variations along manifold line. Thus, inlet head and discharge for all laterals in the system are determined. Also, total inlet discharge of the manifold is estimated.
3. Model verification and system evaluation routines were added.

System evaluation and model verification

The model of analysis described above was utilized in analyzing the hydraulics of 14 trickle irrigation units shown in Table 4. After estimating inlet discharge to manifold, average outlet discharge was estimated dividing inlet discharge to manifold by the number of outlets in the system. As

Table 3. Comparison between polynomial approximation and power relations for outlet equivalents of laterals.

Lateral	Inlet flow l/s	Polynomial relation 7 terms				Power relation 2 terms			
		Absolute errors		Relative errors		Absolute errors		Relative errors	
		Max[1] error ($\times 10^{-3}$)	SASE[2] ($\times 10^{-3}$)	Max error ($\times 10^{-3}$)	SASE ($\times 10^{-3}$)	Max error ($\times 10^{-3}$)	SASE ($\times 10^{-3}$)	Max error ($\times 10^{-3}$)	SASE ($\times 10^{-3}$)
1a	0.4577	0.033	0.0026	0.0728	0.0057	0.017	0.0015	0.038	0.003
1b	0.441	0.263	0.0196	0.596	0.0444	1.260	0.110	2.860	0.249
1c	0.474	0.110	0.0078	0.232	0.0165	0.869	0.0779	1.830	0.160
2a	0.862	0.475	0.0328	0.551	0.0381	0.042	0.00355	0.048	0.004
2b	0.308	0.722	0.0410	2.34	0.1330	8.900	0.076	28.900	2.470
2c	0.404	0.516	0.0420	1.28	0.1040	1.590	0.0142	3.940	0.351
3a	2.700	7.820	0.6470	2.90	0.2400	0.031	0.0024	0.011	0.001
3b	2.560	0.588	0.0476	0.230	0.0186	15.000	1.36	5.860	0.531
3c	2.830	1.380	0.0980	0.488	0.0346	9.900	0.888	3.500	0.314
4a	5.620	1.900	0.1500	0.338	0.0267	0.220	0.0190	0.039	0.003
4b	5.030	1.750	0.1700	0.348	0.0338	61.000	5.50	12.100	1.090
4c	4.920	3.850	0.2600	0.783	0.0528	30.700	2.73	6.240	0.555
5a	0.733	0. 245	0.0210	0.334	0.0286	0.053	0.00447	0.072	0.006
5b	0.620	1.250	0.1070	2.02	0.1730	19.000	1.57	30.600	2.530
5c	0.826	0.168	0.0100	0.203	0.0121	2.780	0.240	3.370	0.291
Aver.	1.919	1.405	0.1100	0.847	0.0641	10.090	0.894	6.631	0.571

[1]Max: Maximum, [2]SASE: square root of the average squared error

inlet discharge and inlet head to each lateral are known from previous analysis, a forward stepwise algorithm is followed along each lateral to estimate discharge from each outlet. The absolute value of the difference between individual outlet discharge and average outlet discharge is estimated. Cumulative additions for the differences in addition to the squares of outlet discharges were done along each lateral. This eliminated the need to store individual outlet discharges and thus reduced memory requirements. After analyzing all the laterals of the system, the sum of the absolute values of differences between outlet discharges and average discharge becomes known and thus Christiansen uniformity coefficient is determined for the unit. From sums of the outlet discharge squared, the average outlet discharge, and the number of outlets, the standard deviations of outlet discharge were estimated. This value is used to estimate coefficient of variations for outlet discharge and thus statistical uniformity coefficient.

As a regression relation describing inlet lateral discharge versus inlet lateral head was utilized, there is an error in estimation. To estimate this error for each lateral in the unit from inlet lateral discharge and inlet lateral head, a forward step method is followed until the end of the lateral. At the last outlet in the lateral, the difference between discharge in the last segment of the lateral and the discharge from that outlet (obtained in eq. 1) represents the error of estimation for that lateral. The absolute values of errors for all laterals in each system were estimated and their summation was determined and shown in Table 5. The computer time utilized in the analysis was also determined (a personal computer with Pentium II, 166 mhz was utilized for all the analyses).

Table 5 shows the results of analyzing 14 trickle irrigation units using the model developed in this study. It shows that errors of estimation were negligible which verifies the use of the model. The model was also successful in evaluating the trickle irrigation systems under consideration through determining statistical uniformity and Christiansen uniformity coefficients. Table 5 shows that time required by a personal computer to run the model is minimal and was less than one second for all the examples shown in this study. This is due to the power of convergence of the Newton Raphson technique and the conversion of laterals into equivalent outlets.

As the computer time needed is low and the model could successfully analyze trickle irrigation systems, the model could be also used in designing trickle irrigation systems. This could be utilized through using the model to evaluate different design alternatives to select the most appropriate one. It could be also modified to design systems according to design requirements which could include: finding maximum length of a manifold that could be used to achieve a certain uniformity, the required inlet hydraulic head to

Table 4. Description of a set of trickle units used in the hydraulic analysis.

System	Lateral direction 1		Lateral direction 2		Manifold		Total
No. of	Slope	No. of	Slope	No. of	Diameter	No. of	
	outlets		outlets		laterals	(mm)	outlets
1	40	0	0	0	20	25.0	800
2	100	0	0	0	20	50.0	2000
3	200	0	0	0	20	65.0	4000
4	40	0	0	0	50	50.0	2000
5	100	0	0	0	50	65.0	5000
6	200	0	0	0	50	75.0	10000
7	40	0	40	0	20	40.0	1600
8	40	0.05	40	–0.05	20	40.0	1600
9	100	0	100	0	20	65.0	4000
10	100	0.05	100	–0.05	20	65.0	4000
11	100	0.05	150	–0.05	20	75.0	5000
12	40	0.05	60	–0.05	50	75.0	5000
13	60	0.0	80	0.0	100	65.0	14000
14	40	0.05	60	–0.05	100	75.0	10000

Other common characteristics:
K=0.0005, X=0.5, Lateral diameter 16.0 mm, Spacing between outlets = 0.5 m, C_H = 140, Spacing between laterals = 2 m, and Manifold slope = 0.0

achieve a required average outlet discharge in the farm, or to test the required diameter for manifold pipes and laterals.

Conclusions

Considering all laterals and outlets in solving the hydraulics of a real field scale trickle irrigation system causes memory problems. This work shows that it is possible to replace lateral lines by equivalent outlets and thus replacing manifold lines by laterals. Although polynomial equations with several parameters could be used to accurately describe relations between inlet discharge and inlet lateral head, these relations could be adequately described by simple power relations similar to those typically used to describe outlet discharge versus outlet head. Such relations with two coefficients are sufficiently accurate for actual field laterals. Newton Raphson method proved to be an efficient method to solve the hydraulics of trickle laterals or manifolds with real size problems. The nature of laterals and manifolds makes it possible to utilize

Table 5. Results of the analysis of trickle units (summary).

System	Inlet flow (l/s)	Head at manifold end (m)	Statistical uniformity %	Christiansen uniformity %	Time needed (seconds)	Sum of errors (l/s) ($\times 10^{-3}$)
1	1.55	13.58	94.50	95.40	0.11	0.002
2	4.20	18.56	98.32	98.60	0.11	0.04
3	7.13	18.927	91.06	92.48	0.17	0.4
4	4.17	16.64	97.37	97.79	0.17	0.005
5	9.82	15.46	96.02	96.73	0.16	0.07
6	16.12	14.40	89.72	91.58	0.22	0.8
7	3.37	17.18	97.92	98.26	0.05	0.004
8	3.37	17.18	97.35	97.88	0.11	2.0
9	8.40	18.55	98.31	98.60	0.11	0.08
10	8.39	18.55	95.76	96.28	0.16	1.0
11	10.30	18.94	97.26	97.87	0.16	1.0
12	10.61	17.35	97.39	97.90	0.16	9.0
13	16.35	2.53	65.11	70.51	0.33	1.0
14	16.88	8.76	87.00	89.20	0.22	3.0

the symmetry and sparse matrix characteristics for the Jacobian matrices used for solving such systems. The nature of laterals and manifolds results in Jacobian matrices with two columns and N rows, where N is the number of outlets (or equivalent outlets for laterals) in the lateral (or manifold) under consideration. This results in significantly reducing memory requirements for solving such systems. The solution required very short computer times (less than one second for each system considered in this study). This makes the solution algorithm efficient in evaluating trickle irrigation systems and thus an efficient tool that could be used in designing trickle irrigation systems.

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