OPTIMIZATION OF WATER SUPPLY NETWORK FOR NABLUS MUNICIPALITY

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ABSTRACT

Water supply of a city is very vital for inhabitants living in it. Effective management of water supply resources is of great importance due to shortage of these resources and the increasing pressures resulting from demographic and/or economic growth and ecological deterioration. Water distribution and storage allocation need to be carefully examined in order to satisfy water demands while minimizing the various types of associated costs. Technical and financial constraints compel the motivation to optimally manage the water supply network (WSN). Nablus city has many limited water resources that have to be optimally managed within a well-controlled system that collects, analyzes, and optimizes data for each demand node through the entire WSN. In this study, we model the drinking water distribution and allocation/usage problem over the entire WSN of Nablus city using Linear Programming (LP). We then optimize the model seeking for optimal water distribution and allocation/usage policies which maximize water availability and minimize the total incurred costs in the supply network. Such optimal policies will aid decision-makers at Nablus municipality in their short-, medium- and long-term planning to secure sufficient quantities with proper qualities of drinking water and to bridge the current and forecasted water supply-demand gap while keeping their technical (capacity) and financial constraints within their feasible limits. Initial solutions of the model resulted in better distribution and allocation policies compared to the current policies adopted by the Nablus municipality. Specifically, the new optimal policies resulted in 35% savings in water pumping costs in the WSN.

Keywords: urban water supply network, linear programming, drinking water, water allocation and usage policy.

1 INTRODUCTION

Word population is increasing rapidly and is forecasted to reach from the 7 billion capita in 2011 to more than 9.3 billion capita by 2050 [1]. This projected increase in population growth will result in a continuous demand on natural resources. Among these resources is drinking water. Securing adequate quantities with proper qualities of drinking water has becoming a big challenge to policy-makers especially with continuous global climatic changes, increasing growth in population and increasing demand on drinking water consumption due to urbanization. Consequently, effective urban (drinking) water supply management and optimization are needed to support policy-makers in resolving the problems associated with water supply distribution and allocation problems.

Many people do believe that if there will be a third world war, it will be on water, especially in countries which experience drought periods, suffer from lack of fresh water resources as well as encounter water-share distribution and allocation problems with neighboring countries. One of these countries is Palestine which has been under Israeli occupation for more than six decades. Israeli occupation is not only limited to land confiscation; it includes confiscation of other natural resources above and under land. Every day Israel seizes fresh water resources in the Palestinian occupied territories. Such resources include fresh water springs and groundwater basins and aquifers.

Palestinian fresh water resources are being depleted by Israeli people in settlements and hence the remaining quantities of fresh water supply do not satisfy the urban water demand requirements of Palestinian citizens. This creates a serious supply, distribution and allocation problem of urban water in all Palestinian cities. Among these is Nablus city, where urban water distribution and allocation are regulated under the supervision of Nablus Municipality. The municipality serves more than 150,000 inhabitants with electricity, water supply, sewage network and other municipal services. More details about Nablus city WSN structure are presented in section 3.1.

The structure of the paper is organized as follows. Section 2 includes the review of some literature pertinent to our study. Section 3 provides the preliminaries and the formulation of the model. Some preliminary solutions and results are presented in Section 4. The last section, Section 5, concludes our work and provides some extensions for future research.

2 LITERATURE REVIEW

Urban water supply networks optimization models were traditionally built on simulation and experiences. However, recently, people have started using mathematical programming models to optimize urban water supply networks. Such models seek for the optimal distribution, usage and allocation of water starting from supply sources to end users to satisfy demand requirements and capacity constraints. Nikjoofar and Zarghami pointed out that many algorithms for minimizing water supply network costs have been employed in various optimization techniques, including linear programming, mixed integer programming, nonlinear programming, dynamic programming, genetic algorithms and global optimization [2].

Many studies have already been dedicated to optimize water distribution, usage and allocation in urban water supply networks. A pretty recent work by Sörensen and De Corte, presents a state-of-art survey of literature done in optimization of water distribution network design (WDND). In WDND optimization problem, optimal solutions for the material and diameter of network pipes in the network are found such that the total cost of the network is minimized while satisfying certain hydraulic constraints. The authors highlighted the main optimization methods, especially in the field of meta heuristics, usually employed in WDND optimization problems [3].

Rozos and Makropoulus developed a new optimization tool called Urban Water Optioneering Tool (UWOT) based on urban metabolism model to simulate the supply and demand strategies and systems within the common modelling environment. Such tool enables policy-makers to simulate the entire urban water cycle from source (supply) to tap (demand) and vice versa. More specifically, the complete network of water supply could be simulated starting from demand generation at end users (households) back to water reservoirs. It also tracks wastewater generation at households through the wastewater system and treatment plants to the water bodies [4]. Another work by Bieupoude *et al.* addresses the drinking water distribution network optimization problem focusing on pipe networks in urban and rural areas [5]. More specifically, the article started with a brief review of computer-based design methods utilized in optimizing drinking water distribution. Then, the authors suggested a complementary geometric analysis approach to be augmented with usual optimization methods where some numerical examples are presented. Many works have been devoted to system performance and rehabilitation strategies for urban water distribution networks. A work by Engelhardt *et al.* presents a literature review with UK perspective on the rehabilitation strategies for water distribution networks [6].

Other works related to water resources management have been done. To name but a few, Songsong *et al.* [7] considered integrated water resources management for water deficient areas considering desalinated seawater, wastewater and reclaimed water. In their paper, a two-step optimization approach has been adopted, where in the first step; the pumping cost was estimated from an approximation model to determine the plant locations, whereas the second step was devoted for solving the entire model, [7]. Seog *et al.* developed a mathematical model to integrate and optimize urban water infrastructures for supply-size planning policy. Particularly, freshwater resources and treated wastewater are allocated to different demand classes with the aim of reducing contaminants available in drinking water supply as well as reducing water resources consumption [8]. Other studies related to urban water distribution management have been done; the reader is encouraged to refer to Zaman and Lehman [9], Eker *et al.* [10] and Gupta and Khanna [11].

3 THE MODEL

3.1 Nablus City Water Supply Network Structure

In the municipality of Nablus city, water resources department is responsible for the management and expansion planning of the urban water network in the city. The network is the main source of water (covering around 95% of citizens in the city) and it utilizes a very complex network. The network can be divided into two main parts; an external network and an internal one. The external network connects the springs and wells to the city main reservoirs, whereas the latter one focuses on distributing the water throughout the city to demand points which are called pressure zones. The focus of this work (so far) is on the optimization of the internal network only, and the whole water distribution system will be studied in future work.

The WSN under study is formed from three different elements: reservoirs, links and demand nodes (pressure zones). A reservoir (*R*) is a main storage of the drinking water. Each reservoir is supplied from either groundwater wells or springs. Typically, reservoirs are built by excavation in the ground or by conservative construction methods such as brickwork or cast concrete. The second element in the internal network is the links which join the network parts together. They are mostly made from concrete cylinders that are designed and selected to transfer amounts of water proportional to the water demand.

Finally, the pressure zones (PZ) are the last element in the network. They represent an area of customers requiring drinking water continuously. Thus, each PZ represented by a demand node that should be satisfied by the municipality's WSN. The network makes it harder to control the distribution process, where many losses occur in the network in each part of it in the form of leakage, thefts, unplanned flow of water between different points. In addition, fulfilment of the customers' need of drinking water with the least pumping cost is considered a massive concern in the water resources management. The members and engineers in the water resources department need a system which guarantees them the satisfaction of the PZ with proper amounts of drinking water at the least cost and losses. The water network of Nablus city encompasses sources of water that feed the entire network which are the reservoirs; there are eleven secondary and primary reservoirs. Each reservoir has its own capacity given in m^3. Each reservoir's capacity and average output per month (m^3) are reported in Table 1. However, since the capacity of the reservoirs rarely affects monthly demand, the reservoirs' capacity (at this stage of the work) will not be taken into account, i.e. the cases where some reservoirs become full are neglected. This assumption is valid and was accepted by the head of the water department at Nablus Municipality.

Table 1 Reservoirs' names, capacities and their average output/month (m^3) based on year 2012

Reservoir Code	**Reservoir Name**	**Max Capacity (m^3)**	**Average Output (m^3 /month)**	**Reservoir Code**	**Reservoir Name**	**Max Capacity (m^3)**	**Average Output (m^3 /month)**
A	RNE4	30,881	27,392	G	Al-Masakin	23,235	24,180
B	Northern Reservoir	29,402	11,552	H	Ein Dafna	173,840	160,310
C	EinBet Elma	76,587	67,126	I	New Reservoir	65,223	58,788
D	Kamal Junblat	70,539	64,554	J	Aseera	44,542	41,932
E	Southern	44,595	35,480	K	Al-Juneed	20,157	16,845
F	Al-Sumara	44,929	41,473				

Source: Water resources management department, Nablus Municipality

In addition, the network has PZ that represents demand nodes. The existing WSN has 25 pressure zones. Table 2 shows the names of the pressure zones and their average monthly demand according to collected data for the year 2012, as reported by the water resources management department at Nablus Municipality

Table 2 The pressure zone names and average monthly demand in Nablus City for year 2012

Number	**Location**	**Avg. Monthly Demand (m^3)**	**Number**	**Location**	**Avg. Monthly Demand (m^3)**
1	NE1	21644	14	S5	16253
2	NE2	28098	15	E1	21771
3	NE3	25829	16	E2	35897
4	NE4	1540	17	E3	35440
5	W0	22663	18	SE1	21226
6	W1	22771	19	SE2	9548
7	W-1	23080	20	SE3	4355
8	W2	64747	21	C1	60112
9	W3	40565	22	NW0	11663.8
10	W4	19448	23	NW1	6931
11	S2	29663	24	NW2	16853
12	S3	14507	25	NW3	8800
13	S4	13207			

3.2 Model Formulation

The notation and the formulation of the proposed mathematical model are as follows

Indices

i	Reservoir (R) index
j	Pressure Zone (PZ) index

Parameters

NR	Number of Reservoirs
NPZ	Number of Pressure Zones
NRPZ	Total Number of possible connections between Pressure Zones and Reservoirs; which is equal to NR×NPZ
c_{ij}	The cost of pumping one cubic meter of water from Reservoir i to Pressure Zone j
MQ_i	The maximum quantity pumped from Reservoir i during one time period (one month).
DEM_j	The demand of Pressure Zone j
Z	Total water distribution cost in the network

Decision Variable

x_{ij}	The quantity pumped (in cubic meters) from Reservoir i to Pressure Zone j

Objective Function

$$\text{Min } Z = \sum_i^{NR} \sum_j^{NPZ} c_{ij} x_{ij} \qquad (1)$$

Subject to

$$\sum_i^{NR} x_{ij} \geq \mathrm{DEM}_j \quad \text{for all } j = 1,2,\ldots.\mathrm{NPZ} \qquad (2)$$

$$\sum_j^{NPZ} x_{ij} \leq \mathrm{MQ}_i \quad \text{for all } i = 1,2 \ldots.\mathrm{NR} \qquad (3)$$

$$x_{ij} \geq 0 \quad \text{for all } i = 1,2,\ldots\mathrm{NR} \quad \text{and} \quad j = 1,2,\ldots.\mathrm{NPZ} \qquad (4)$$

Eq. (1) states the objective of this model as to minimize the total water distribution cost. Eq. (2) indicates that the summation of all quantities received at a certain PZ should be at least equal to water demand received on that PZ. Eq. (3) sets the maximum quantity a reservoir (R) can distribute (based on the data of the year 2012). Eq. (3) is needed as the proposed model only focuses on the internal WSN, whereas pumping quantities from the wells and springs to the reservoirs are excluded. Eq. (4) ensures that pumping quantities should be larger or equal to zero.

To be able to solve this model to obtain the optimal distribution quantities (x_{ij}), the cost per cubic meter pumped from reservoir i to pressure zone j (c_{ij}) should be estimated. Namely, c_{ij} depends on the amount of electrical power consumed by pumps per cubic meter of water being pumped. Mathematically, this is given by Eq. (5) below:

$$c_{ij} = \frac{9.81H_{ij} \times Q_{ij} \times \psi}{1000\eta_i} \tag{5}$$

Where:

H_{ij} is the differential head (meters) between the reservoir *i* and pressure zone *j*,
Q_{ij} is the flow capacity in the pipes between reservoir *i* and pressure zone *j*,
ψ is the cost of one KW-h of electricity,
η_i is the overall pump efficiency of reservoir *i*.

Clearly, c_{ij} depends on the flow capacity of the links (the pipes), the differential head (the difference in the altitudes) and pump's overall efficiency. Thus, the cost matrix (from each reservoir to each pressure zone) can be easily constructed. The cost matrix relevant to Nablus city urban water distribution network is given in the Appendix. It should be noticed that if the connection is not possible from reservoir *i* to pressure zone *j*, the corresponding unit cost would be set to a relatively-very high value (e.g. 1000) and, thus, the optimization algorithm will avoid using it. On the other hand, if the altitude of the reservoir is much higher than the pressure zone, no pumping is needed, and gravitational flow of water is allowed. Thus, the cost associated with this situation is zero.

3.3 Current Situation

The current water distribution quantities adopted by the water department are based on experience and simple heuristic methods that depend on proximity/altitude difference between the reservoir and the target pressure zone. Exact distribution costs can be calculated applying the cost calculation approach seen in Eq. (1), that is

$$\text{Total pumping cost} = \sum_{i}^{NR} \sum_{j}^{NPZ} c_{ij} x_{ij} \tag{6}$$

Table 3 shows the average distribution quantities of the year 2012, as obtained from the water resources management department. It is obvious that the total number of links (connections between reservoirs and pressure zones) is limited to 26. The associated direct total monthly pumping cost, using Eq. (6), is equal to 97,723 NIS.

Table 3 The Adopted distribution quantities for year 2012

Reservoir (*i*)	**Pressure Zone (*j*)**	**Pumped Quantity x_{ij} (m³)**
RNE4	NE3	25994
	NE4	1399
Ein Bait Elma	W0	22663
	W1	22771
	W-1	23080
Northern	NE2	28098
Junblat	W2	64747
Southern	S3	14507
	S4	13207
	S5	16253
Al-Sumara	W3	40565
Al-Masakin	E1	21771
New Res.	C1	45683
Ein Dafna	NE1	21645
	S2	29663
	E2	35897
	E3	35440
	SE1	21226
	SE2	9548
	SE3	4335
	C1	14429
Aseera	NW0	11664
	NW1	6931
	NW2	16853
	NW3	8800
Junaid	W4	19448

4 PRELIMINARY RESULTS

To solve the above-formulated linear programming (LP) problem, the model was programmed using ZIMPL Mathematical Programming Language [12], and a free open source optimization program lp_solve [13]. The code is given in the Appendix.

The total cost obtained after solving the optimization problem was equal to 63,588 NIS/ month, about 35% lower than cost associated with the current situation. Table 4 shows the new allocation quantities obtained from the solution of the LP model.

Table 4 Optimal water distribution quantities for each reservoir (m^3/month)

Reservoir (*i*)	**Pressure Zone (*j*)**	**Pumped Quantity x_{ij} (m^3)**
RNE4	NE3	25829
	NE4	1540
Ein Bait Elma	W0	22663
	W1	22771
	W-1	23080
Northern	NE2	28098
Junblat	W2	64747
Southern	W3	21115
	W4	19448
Al-Sumara	S4	10410
	S5	16253
Al-Masakin	E1	21771
New Res.	SE1	19426
	S2	26258
Ein Dafna	S3	14507
	S4	2797
	E2	35897
	E3	35440
	NE1	21644
	SE1	1800
Aseera	NW0	11664
	NW1	6931
	NW2	16853
	NW3	8800
Junaid	W3	19450

5 CONCLUSIONS AND FUTURE EXTENSIONS

This work presents a practical application of optimization methods; specifically linear programming (LP), in the area of urban water distribution networks. The aim of this work is to find the optimal distribution quantities of drinking water in Nablus city’s distribution network which minimize the total monthly water pumping cost in the network. The new distribution quantities rendered by the model resulted in average monthly pumping cost by about 35%. The LP optimization method proved to be effective in finding optimal pumping quantities and it should be considered while designing the urban water distribution network.

It is worth pointing here that the present work is an ongoing works and it is expected to be extended to augment other aspects of the network on the model. In particular, we first plan to include the whole water distribution network (considering water sources such as wells and springs) in the model. This would give a more comprehensive solution, and can be used for network operation and expansion planning purposes. Another aspect is to find the distribution water quantities per month of the year; this will convert the model to a dynamic program and probably would reduce the total pumping cost more if implemented correctly. We do believe that such extensions on the current model will result in a more realistic model and eventually more practical insights and solutions to the policy-makers.

6 ACKNOWLEDGEMENT

The authors would like to thank the staff of the Water Resources Management Department at Nablus Municipality for their cooperation in facilitating the conduction of this work.

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8 APPENDIX: lp_solve 5.5 SOFTWARE CODE

```
# LPSolver
```

```
 set Reservoir:={"A","B","C","D","E","F","G","H","I","J","K"};
 set  PZ:={"NE3","NE4"  , "NE2" , "W1" , "W0",   "W-1"  , "W2"   ,"S3"   , "S4"  , "S5" , "W3"    , "W4"  , "E1" , "E2"
,"E3" ,   "NE1"  ,"SE1" , "SE2", "S2"  ,"SE3" , "C1"  , "NW0", "NW1", "NW2"   ,  "NW3"}  ;
 set RPz:= Reservoir*PZ   ;
# parameter definitions
  param demand [PZ]:= <"NE3"> 21645,  <"NE4"> 28098 ,  <"NE2"> 25829  ,  <"W1"> 1540,  <"W0"> 22663  ,
             <"W-1"> 22771  ,  <"W2"> 23080  ,  <"S3"> 64747  , <"S4"> 40565 , <"S5"> 19448 ,
              <"W3"> 29663 , <"W4"> 14507  ,  <"E1"> 13207 ,   <"E2"> 16253,  <"E3"> 21771 ,
              <"NE1"> 35897,  <"SE1"> 35440 ,  <"SE2"> 21226  , <"S2"> 9548 ,  <"SE3"> 4355 ,
              <"C1"> 60112 , <"NW0"> 11664   ,  <"NW1"> 6931   ,  <"NW2"> 16853  , <"NW3"> 8800 ;
    param MQ [Reservoir]:= <"A">  27369 , <"B"> 28100 , <"C"> 68515 , <"D"> 64750 , <"E"> 43968 ,
             <"F"> 40566 , <"G"> 21772 , <"H"> 172202 , <"I">  45684 , <"J"> 44248, <"K"> 19450 ;

# variable definitions
 var x[RPz] real; #quantity pumped from Reservoir to PZ
 param cost [RPz]:=
 |"NE3","NE4"  , "NE2" , "W1" , "W0",   "W-1" , "W2"   ,"S3"   , "S4"  , "S5" , "W3"    ,    "W4"  , "E1" , "E2"
           ,"E3" ,   "NE1"  ,"SE1" , "SE2", "S2"  ,"SE3" , "C1"  , "NW0", "NW1", "NW2"  ,  "NW3"|
|"A"|  0, 1000,   0,       .186,    0,       1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,
          1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000 |
|"B"|  1000,       0,       1000,    1000,    0,       .4,      1000,    1000,    1000,    1000,    1000,    1000,
          1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,
          1000          |
|"C"|  1000,       1000,    1000,    1000,    .027,    .09,     .1889,   1000,    1000,    1000,    1000,    1000,
          1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,    0,       .1944,   1000,
          1000          |
|"D"|  1000,       1000,    1000,    1000,    .305,    .167,    1000,    .055,    .166,    1000,    1000,    1000,
          1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,
          1000          |
|"E"|  .433,       1000,    1000,    .18,     0,       0,       0,       60,      0,       .16,     0,       0.226,
          0.335,   0.528,   .63,     .63,     .63,     .453,    .26,     .09,     0,       0,       0,       0.2 |
|"F"| 0.11, 1000, 1000, 0.33, 0,      0, 0, 0, 0.218, 3.3, 0.1260, 0.15,      0.25, 0.4, 0, 0, 0,0.13, 0.11, 0.15,    .26,
          0,  0,    0.12, 0.13 |
|"G"|  1000,       1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,
          1000,    1000,    .16,     1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,
          1000  |
|"H"|  .2479,      1000,    .353,    .505,    1000,    1000,    1000,    1000,    1000,    1000,    .395,    .311,
          .422,    .603,    0.286,   .0844,   .0899,   .2452,   .39,     .577,    .1,      1000,    1000,    1000,
1000    |
|"I"|  0.176,  0.18, 0.33, .486, 0, 0, 1000,      0.1, 0.28,        .417,    .153,    .22,     .416, 0.6,1000,    1000,
          1000,    .138,    .195,    .32,     .245,    1000,    1000,    1000,    1000      |
|"J"| 1000,        1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,
          1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,    0,  0.0899,  0.2534, 0|
|"K"|  1000,       1000,    1000,    1000,    1000,    1000,    1000,    1000,    0,  0.4305, 1000,  1000,    1000,
          1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000,    1000   |;

# Objective function
minimize cost : sum <R,p> in RPz :cost[R,p]*x[R,p];

subto demand_cons:
forall <p> in PZ  do
sum <R> in Reservoir: x[R,p] >= demand [p];

subto max_MQ_cons:
forall <R> in Reservoir do
sum <p> in PZ: x[R,p] <= 1.00* MQ [R]     ;

subto max_MQ_consr:
forall <R,p> in RPz do   x[R,p] >=0;
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