

Sameer M. Ikhdair · Majid Hamzavi

# Relativistic New Yukawa-Like Potential and Tensor Coupling

Received: 26 June 2012 / Accepted: 14 July 2012 / Published online: 1 August 2012  
© Springer-Verlag 2012

**Abstract** We approximately solve the Dirac equation for a new suggested generalized inversely quadratic Yukawa potential including a Coulomb-like tensor interaction with arbitrary spin-orbit coupling quantum number  $\kappa$ . In the framework of the spin and pseudospin (p-spin) symmetry, we obtain the energy eigenvalue equation and the corresponding eigenfunctions, in closed form, by using the parametric Nikiforov–Uvarov method. The numerical results show that the Coulomb-like tensor interaction,  $-T/r$ , removes degeneracies between spin and p-spin state doublets. The Dirac solutions in the presence of exact spin symmetry are reduced to Schrödinger solutions for Yukawa and inversely quadratic Yukawa potentials.

## 1 Introduction

Relativistic symmetries of the Dirac Hamiltonian had been discovered many years ago. However, these symmetries have been recently recognized empirically in nuclear and hadronic spectroscopies [1]. Within the framework of Dirac equation, the p-spin symmetry used to feature deformed nuclei and the superdeformation to establish an effective shell-model [2–4]. The spin symmetry is relevant to mesons [5] and occurs when the difference of the scalar  $S(r)$  and vector  $V(r)$  potentials are constant, i.e.,  $\Delta(r) = C_s$  and the p-spin symmetry occurs when the sum of the scalar and vector potentials are constant, i.e.,  $\Sigma(r) = C_{ps}$  [6,7]. The p-spin symmetry refers to a quasi-degeneracy of single nucleon doublets with non-relativistic quantum number  $(n, l, j = l + 1/2)$  and  $(n - 1, l + 2, j = l + 3/2)$ , where  $n$ ,  $l$  and  $j$  denote the single nucleon radial, orbital and total angular quantum numbers, respectively [8,9]. Further, the total angular momentum is  $j = \tilde{l} + \tilde{s}$ , where  $\tilde{l} = l + 1$  pseudo-angular momentum and  $\tilde{s}$  is p-spin angular momentum [10]. Recently, the tensor potentials were introduced into the Dirac equation with the substitution  $\vec{p} \rightarrow \vec{p} - im\omega\beta.\hat{r}U(r)$  and a spin-orbit coupling is added to the Dirac Hamiltonian [11,12]. Lisboa et al. [13] have studied a generalized relativistic harmonic oscillator for spin-1/2 particles by considering a Dirac Hamiltonian that contains quadratic vector and scalar potentials together with a linear tensor potential under the conditions of p-spin and spin symmetry. Alberto et al. [14] studied the contribution of the isoscalar tensor coupling to the realization of p-spin symmetry in nuclei. Akçay [15] solved exactly the Dirac equation with scalar and vector quadratic potentials including a Coulomb-like tensor potential. He also solved exactly the Dirac equation for a linear and Coulomb-like term containing the tensor potential too [16]. Aydoğdu and Sever [17] obtained exact solution of the Dirac equation for the pseudoharmonic potential in the presence of linear tensor potential under the p-spin symmetry and showed

---

S. M. Ikhdair  
Physics Department, Near East University, 922022 Nicosia, North Cyprus, Mersin 10, Turkey  
E-mail: sikhair@neu.edu.tr

M. Hamzavi (✉)  
Department of Basic Sciences, Shahrood Branch, Islamic Azad University, Shahrood, Iran  
E-mail: majid.hamzavi@gmail.com