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Role of Angular Impulse Exerted on a Moving Electron in a Magnetic field

دور الدفع الزاوي المؤثر على الكترون متحرك في مجال مغناطيسي

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Abstract

In this paper, the dynamics of a moving electron in a perpendicular magnetic field region is examined. It is shown that the angular impulse plays an important role in determining the change in the electron's speed when considering the dynamics of motion in two situations. It is shown that, in the first case, the electron will leave its circular orbit with a speed equaling half its orbital initial value when the magnetic field is turned off. In the second situation, the final orbital speed of an electron entering a perpendicular magnetic field is twice its initial speed. The present treatment is suitable for undergraduate students as it deepens their understanding of the dynamics involved in electromagnetic systems.

Key Words: Electromagnetism

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ملخص

في هذا البحث، سيتم دراسة ديناميكا حركة الكترون متحرك في مجال مغناطيسي. سيُرى ان الدفع الزاوي يلعب دوراً هاماً في تحديد التغير في سرعة الالكترون عند النظر في ديناميكا الحركة في حالتين: في الحالة الأولى، سيرى ان الالكترون سيترك مداره بسرعة تساوي نصف سرعته الابتدائية في المدار عندما يتناقص المجال المغناطيسي الى الصفر. أما في الحالة الثانية،

فسيرى أن سرعة الالكترون النهائية في المدار، عند دخوله مجال مغناطيسي عمودي على اتجاه سرعته الابتدائية، تساوي ضعف سرعته الابتدائية قبل دخوله للمجال المغناطيسي. إن المعالجة الحالية تعمق فهم ديناميكا الحركة في الأنظمة الكهرومغناطيسية.

Introduction

The problem of a moving electron entering a perpendicular magnetic field is widely well-known in most introductory physics textbooks [1, 2] and has been examined in different settings in the literature [3-6]. The usual formulation of the problem goes as follows: When a charged particle moving with constant velocity enters a perpendicular magnetic field, it will move in a circular path due to the magnetic force exerted on it by the field. Since this magnetic force is perpendicular to both the velocity and magnetic field, the particle will move in a circle. The usual assertion is that the kinetic energy of the particle remains the same as before it enters the magnetic field since the magnetic force does no work on the particle whose path is perpendicular to the magnetic field. It is true that the magnetic force does no work, but the treatment is over simplified and the dynamics of the problem has not been examined. Therefore the present author is compelled to shed some light on the dynamics involved in this problem. In this paper, we investigate the angular impulse and its effect on the speed of a circulating electron in a perpendicular magnetic field for two situations. The first deals with the dynamics involved when a circulating electron in a perpendicular magnetic field, and at some moment the magnetic field, is switched off. The second deals with the dynamics involved when a moving electron enters a perpendicular magnetic field and starts to circulate in its path. For each situation, we derive the angular impulse exerted on the electron which then determines the final angular momentum.

The dynamics of a circulating electron when the magnetic field is turned off.

Consider an electron moving in the $+\hat{\varphi}$ direction with a speed v_0 in a circular orbit in the x-y plane which is perpendicular to a uniform magnetic field $\vec{B}$ in the z direction. The balance between the magnetic force and the centripetal force yields

$$mv_0 = erB, \tag{1}$$

where m and e are the mass and magnitude of the charge of the electron respectively, r is the radius of the orbit and B is the magnetic field. If the magnetic field is turned off at a rate dB/dt, then an induced electric field will appear which, by Faraday's law of induction, is given by

$$\vec{E} = -\frac{r}{2}\frac{dB}{dt}\hat{\varphi}. \tag{2}$$

Due to this electric field, an electric force will be exerted on the electron whose direction is in the same direction of the electron. The torque of this force about the z axis is given by

$$\vec{\tau} = \vec{r}x\vec{F} = \frac{er^2}{2}\frac{dB}{dt}\hat{z}. \tag{3}$$

The angular impulse $\vec{I}$, imparted by the above torque to the electron is

$$\vec{I} = \int \vec{\tau}\, dt = \frac{er^2}{2}\int_B^0 \frac{dB}{dt}dt = -\frac{er^2}{2}B\,\hat{z}, \tag{4}$$

which, upon the use of Equation (1) becomes

$$\vec{I} = -\frac{1}{2}mv_0 r\,\hat{z}. \tag{5}$$

The above angular impulse is equal to the change in the angular momentum $\Delta\vec{L}$ of the electron. Using the initial angular momentum, $\vec{L}_i = mv_0 r\,\hat{z}$, we thus have

$$\Delta\vec{L} = -\frac{1}{2}mv_0 r\,\hat{z} = \vec{L}_f - \vec{L}_i = \vec{L}_f - mv_0 r\,\hat{z}, \tag{6}$$

which immediately gives

$$\vec{L}_f = \frac{1}{2} m v_0 r \, \hat{z} . \tag{7}$$

The result in Equation (7) shows that the angular impulse slows down the electron and its final speed is half its initial value, *i.e* $v_f = v_0 / 2$. This result can also be achieved by calculating the work done, *W*, on the electron by the induced electric field which is

$$W = -e \oint \vec{E} \cdot d\vec{S} = + \frac{er}{2} \int_B^0 \frac{dB}{dt} dS . \tag{8}$$

Let $\bar{v}$ be the average speed of the electron while the magnetic field is decreasing to its final zero value, so that

$$\frac{dS}{dt} = \bar{v} = \frac{1}{2}(v_0 + v_f) , \tag{9}$$

where v_f is the final speed of the electron with which it will leave its orbit when the magnetic field vanishes. Upon substituting Eq. (9) into Eq. (8) and using Eq. (1) gives

$$W = -\frac{1}{4} m (v_0^2 + v_0 v_f) . \tag{10}$$

The above negative work is equal to the change in the kinetic energy of the electron as demanded by the work-energy theorem, and thus equating this work to $\frac{1}{2m}(v_f^2 - v_0^2)$, one immediately gets

$$2v_f^2 + v_0 v_f - v_0^2 = 0 , \tag{11}$$

whose roots are $v_f = \frac{v_0}{2}$ and $-v_0$. The second root is not consistent with the change of angular momentum given in Eq. (6) and thus must be rejected. The first root is exactly equal to the final speed predicted by the angular impulse. With this value of the final speed, the work done by the induced electric field is -3/4 of the initial kinetic energy and thus the final kinetic energy is 1/4 of the initial value.

The dynamics of a moving electron entering a magnetic field

We consider an electron moving with an initial velocity v_0 entering a perpendicular uniform magnetic field whose direction is along the z axis. The magnetic force exerted on the electron makes the electron move in a circle, which intern encloses a magnetic flux. This means that there is a time rate of change of area which changes from initial value zero to a final one given by the area of the circular path. This is a two-step process and our purpose here is to examine the dynamics of the electron's motion during this process. Therefore, there is a time rate of change of the enclosed magnetic flux and thus an induced electric field will be created along the path of the electron. This change of flux underlines the dynamics involved in the problem which is not taken seriously in the literature. The two-step process just mentioned is equivalent to the situation where a circular current loop is placed in an external magnetic field which is normal to its plane. Of course, in this analog situation, an induced electric field will be created due to a change in the enclosed magnetic flux which results form a change in the area. Therefore, the induced electric field is

$$\vec{E} = -\frac{B}{2\pi r}\frac{dA}{dt}\hat{\varphi}\,, \tag{12}$$

where A is the area enclosed by the circular path. The angular impulse imparted to the electron is just the change in angular momentum and thus we have

$$\vec{I} = \Delta\vec{L} = \int \vec{r}\ x\,(-e\vec{E})dt = \frac{eB}{2\pi}\int_0^A \frac{dA}{dt}dt\,\hat{z} = \frac{eBr^2}{2}\hat{z}\,, \tag{13}$$

where we used $A = \pi\, r^2$. The final speed of the electron satisfies

$$erB = mv_f\,, \tag{14}$$

and thus Eq. (13) becomes

$$\Delta\vec{L} = \vec{L}_f - \vec{L}_i = \frac{mrv_f}{2}\,. \tag{15}$$

The initial and final angular momenta are given by

$$\vec{L}_i = mrv_0\,,\ \vec{L}_f = mrv_f\,, \tag{16}$$

and therefore Eq.(15) immediately yields

$$v_f = 2v_0\,. \tag{17}$$

This means that the final speed of the electron is twice its initial value which is attributed to the positive angular impulse (and thus positive $\Delta\vec{L}$) given to the electron.

Again our result in Eq. (17) can also be derived by considering the work done on the electron by the induced electric field. Using Eq.(12), we have

$$W = \int -e\vec{E}\cdot d\vec{s} = +\frac{eBr}{2}\bar{v}\,, \tag{18}$$

which, by using Eq.(14) and writing $\bar{v} = \frac{1}{2}(v_0 + v_f)$, yields

$$W = \frac{1}{4}m(v_f^2 + v_0 v_f)\,. \tag{19}$$

The above work is just the change in the kinetic energy of the electron, $\frac{1}{2}m(v_f^2 - v_0^2)$, and thus we have the quadratic equation

$$v_f^2 - v_0 v_f - v_0^2 = 0\,, \tag{20}$$

whose solution, which is consistent with the change of angular momentum, is $v_f = 2v_0$.

This gives an increase in the kinetic energy equaling three times the initial value. We must emphasize that the change in kinetic energy must be supplied by source of the magnetic field.

Conclusion

In this paper we have examined the dynamics involved in the problem of a moving electron in a perpendicular magnetic field in two cases. In the first case, an electron steadily circulates in a perpendicular magnetic field with an orbiting speed v_0. When the magnetic field decreases to zero, the final speed of the emerging electron will be half its initial value. This is caused by the angular impulse exerted on the electron by the induced electric field created during the change of the magnetic field. This also can be achieved by the work-energy theorem in which a negative work is done on the electron by the induced field and thus the electron will lose kinetic energy. In the second case, an electron moving with an initial velocity v_0 enters a perpendicular magnetic field. Once the electron starts to circulate around the magnetic field there is a change in area which encloses the magnetic field and thus an induced electric field is created. The angular impulse exerted on the electron increases its speed to twice its initial value. The work done on the electron by the electric force is now positive and thus the work-energy theorem yields an increase in the kinetic energy with a final speed twice its initial value. Therefore, our results show that the final speed of the orbiting electron will decrease to half its initial value for the first situation and will increase to twice its initial value for the second situation.

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